

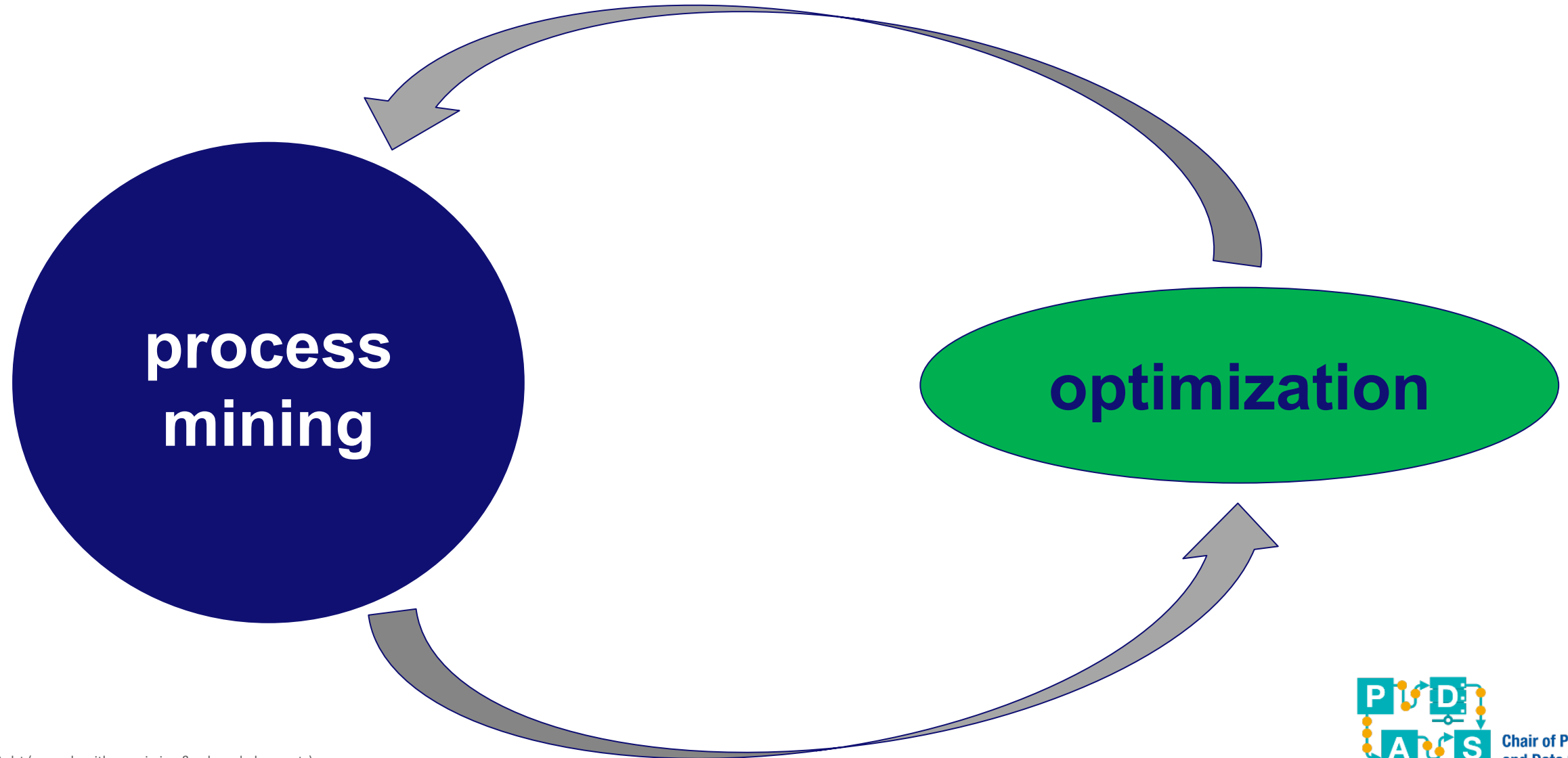
# Process Mining for Optimization and Optimization for Process Mining

Wil van der Aalst  
Process and Data Science @ RWTH  
Aachen University & Celonis

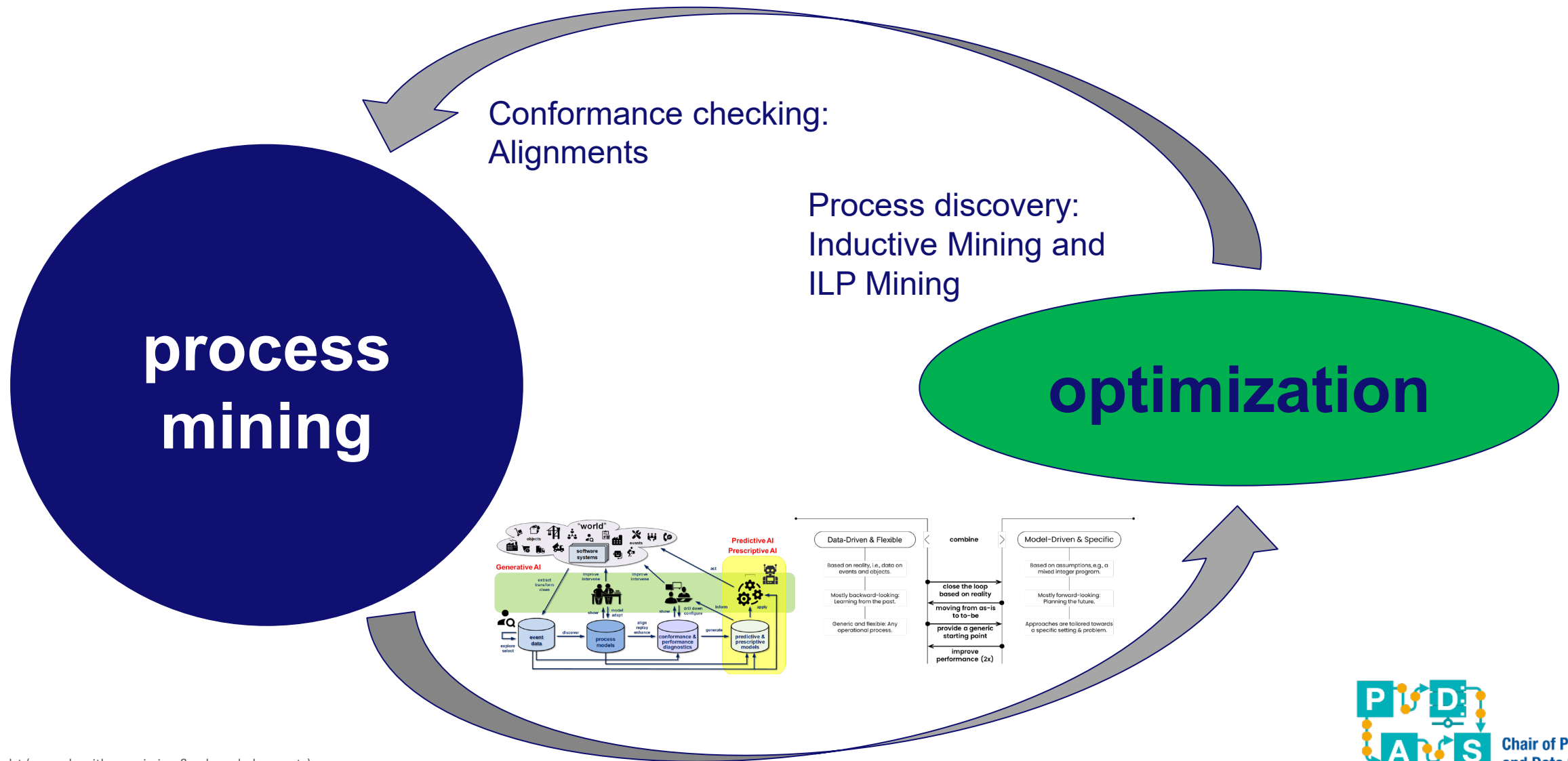


POWERED BY  
PROCESS MINING

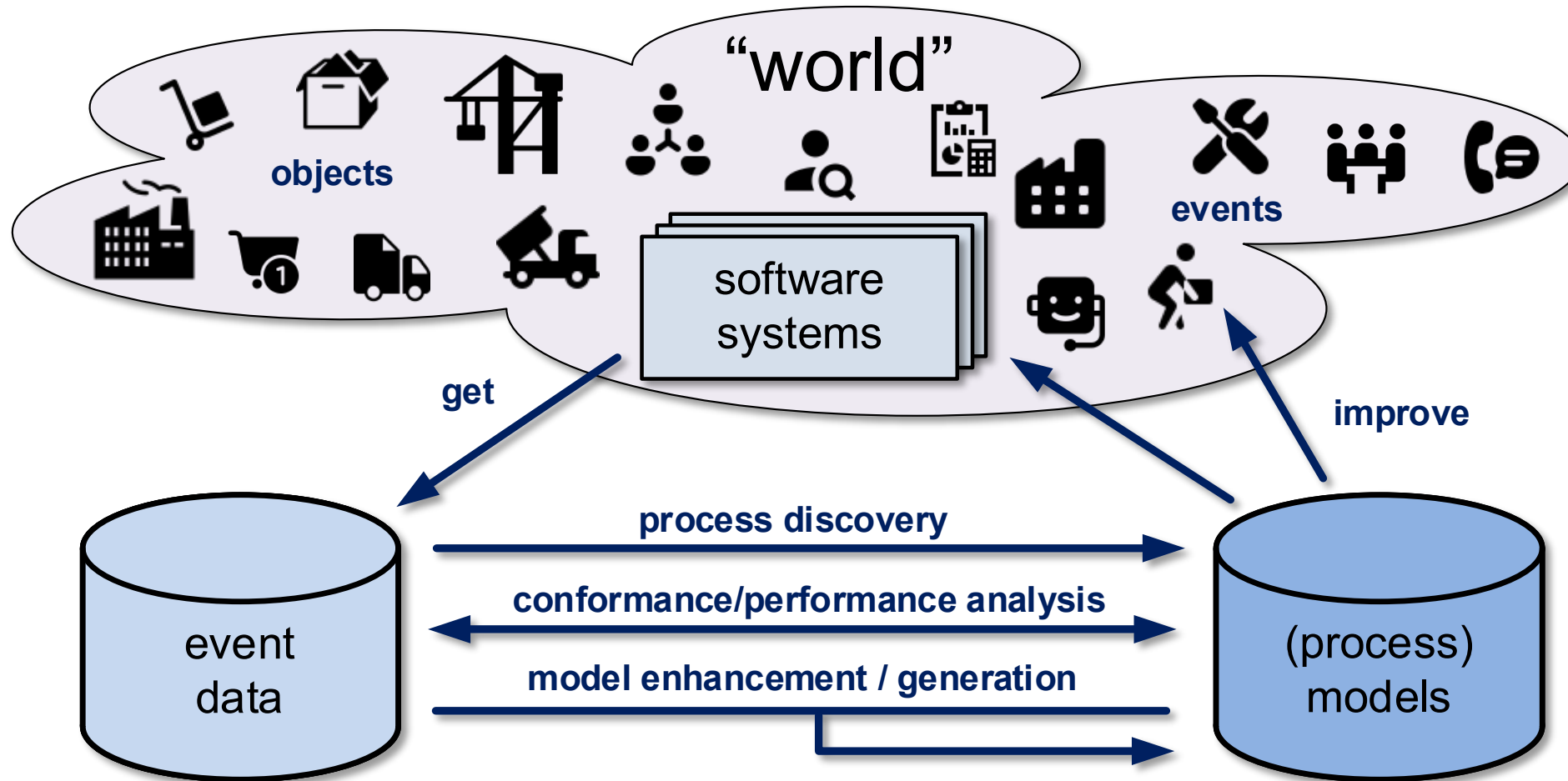
# Outline



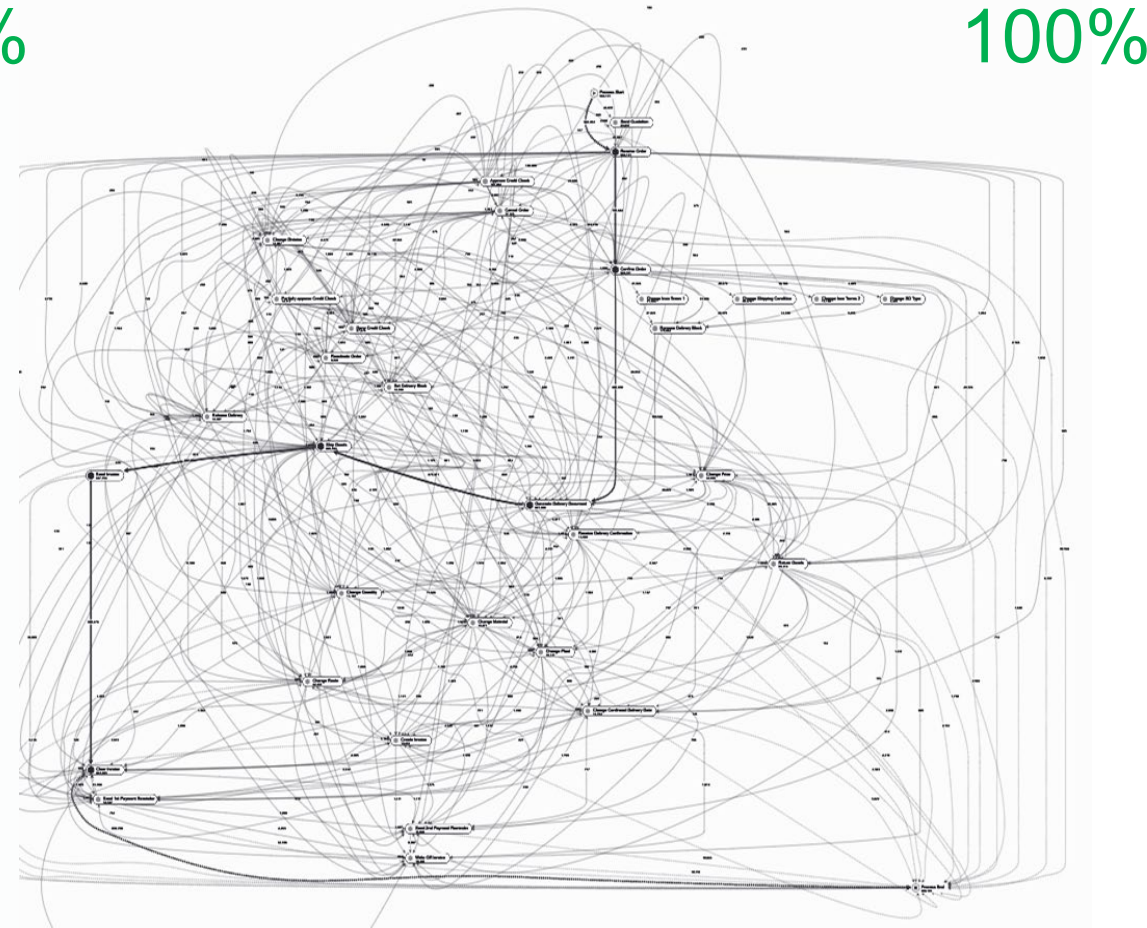
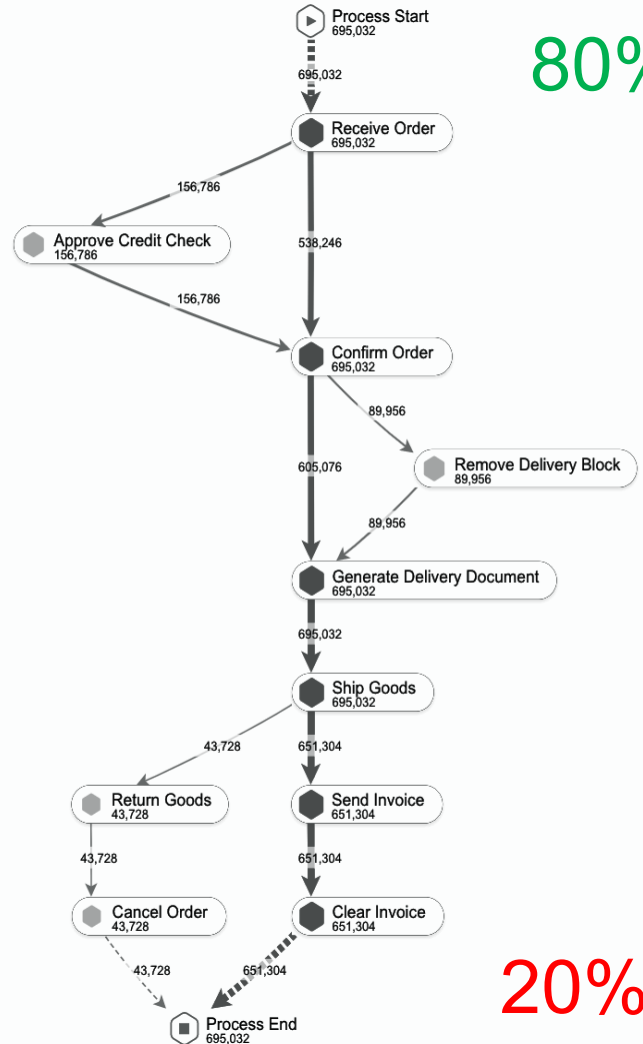
# Outline



# Process Mining: Overview



# Actual processes are very different from what stakeholders think !



# Use Cases Everywhere!

## Technology



## Financial Services & Insurance



## Life Sciences & Chemicals



## Consumer & Retail



## Manufacturing



## TeleCo & Media



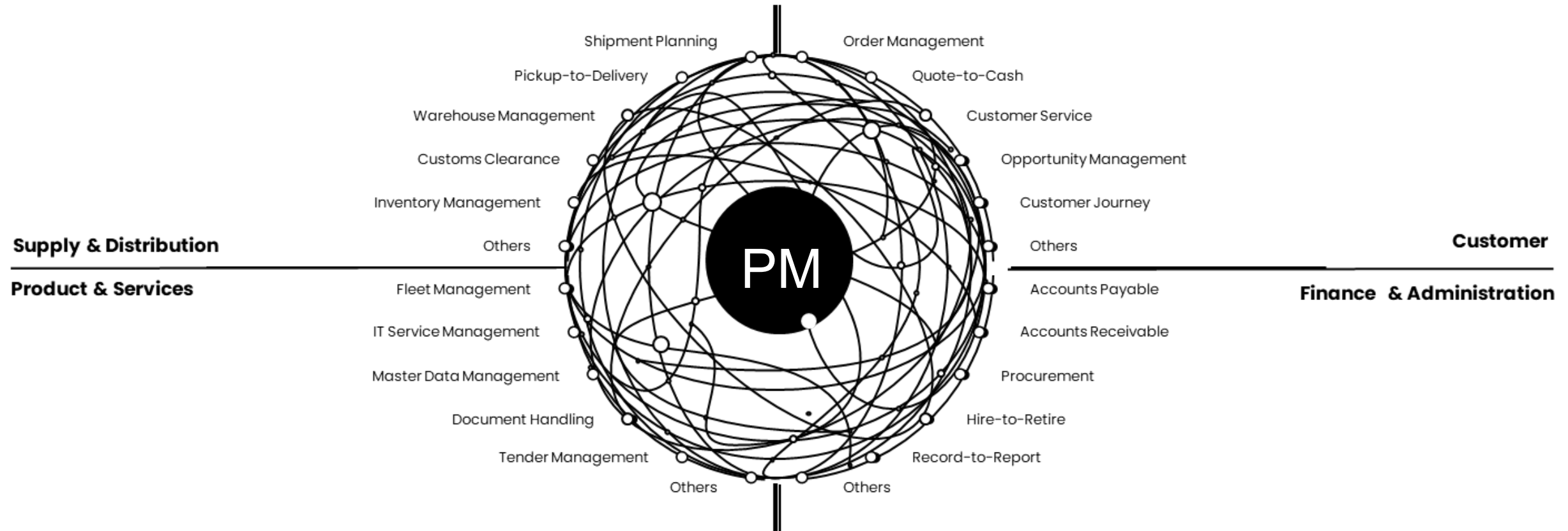
## Energy & Utilities



## Oil & Gas



# Use Cases Everywhere!



**INFORM**

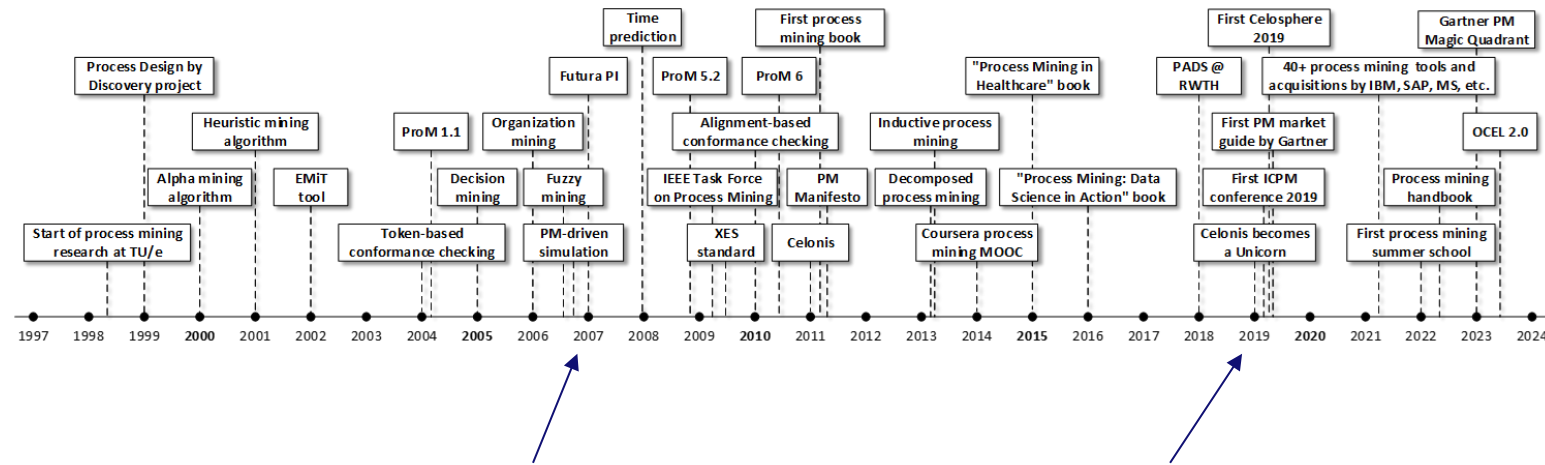
**celonis**



# A new category of tools and scientific discipline ...

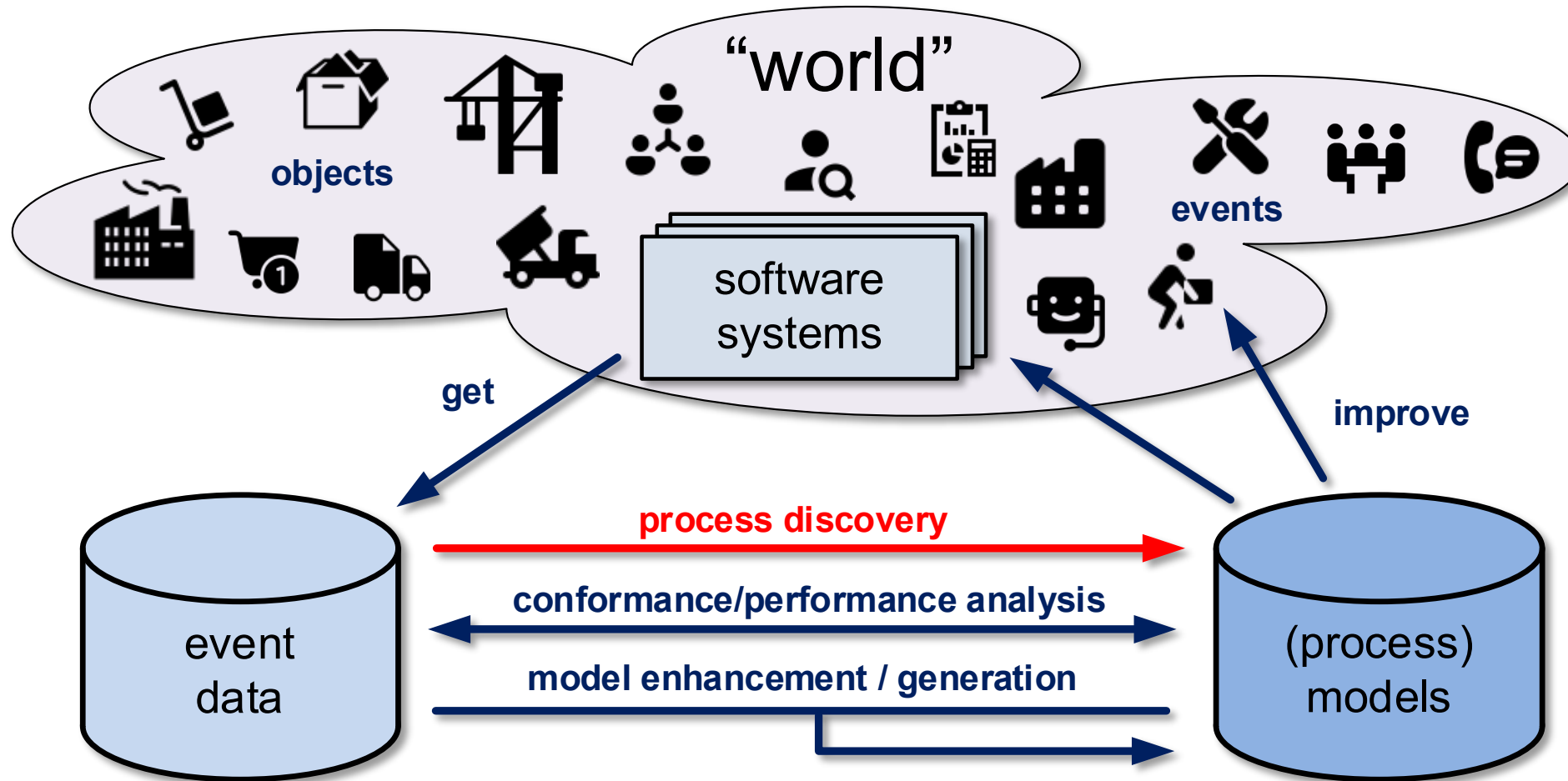


**Gartner Magic Quadrant**

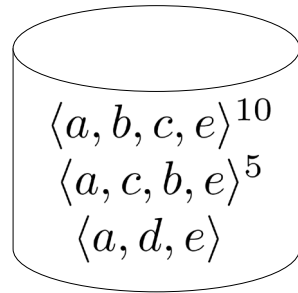


# Example: Process Discovery Using IM

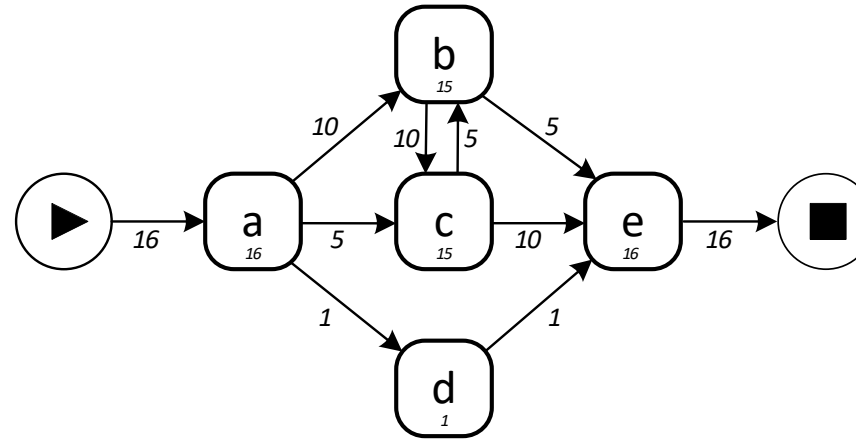
# Process Discovery



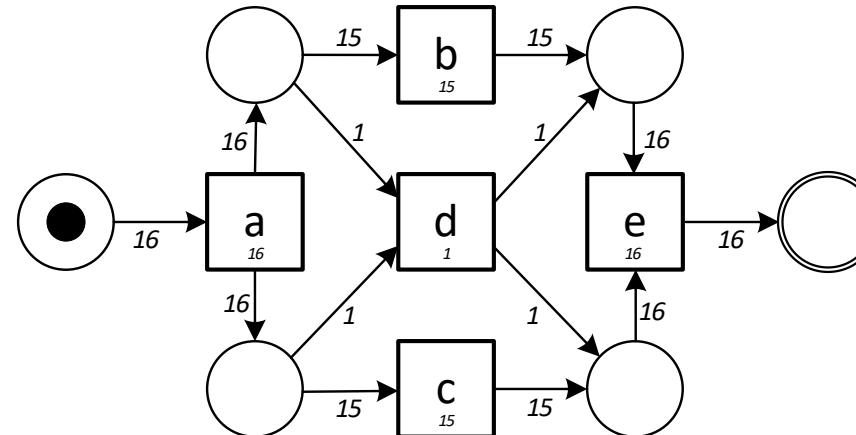
# The main idea (informal)



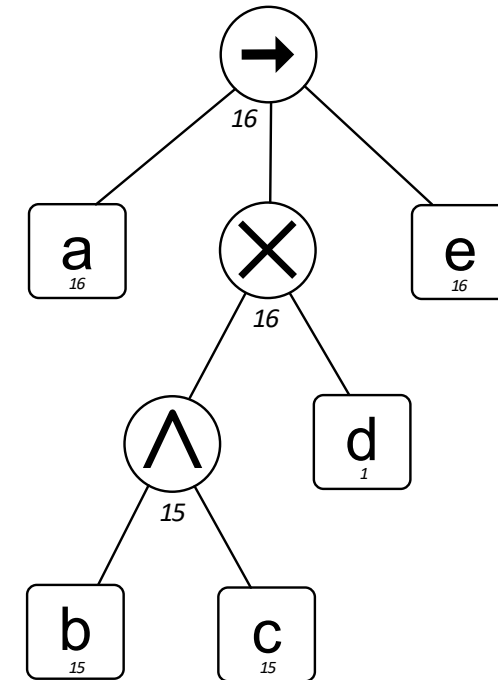
(a) Event log  $L_1$



(b) Directly-Follows Graph (DFG):  $M_1$



(c) Accepting Petri Net (APN):  $M_2$



(d) Process Tree (PT):  $M_3$

# Inductive Mining (IM)

- Decompose the event log into smaller events logs until the problem get trivial.
- Four types of cuts corresponding to the operators:  
→ (sequential composition), × (exclusive choice), ∧ (parallel composition), and ↺ (redo loop).
- In each step the activities are partitioned into subsets until they are singletons.

Developed by Sander Leemans in the context of his PhD thesis (NWO project “Don't Search for the Undesirable! Avoiding “Blind Alleys” in Process Mining” 2012-2017).



**Presented at high speed: You only need to get the main idea!**

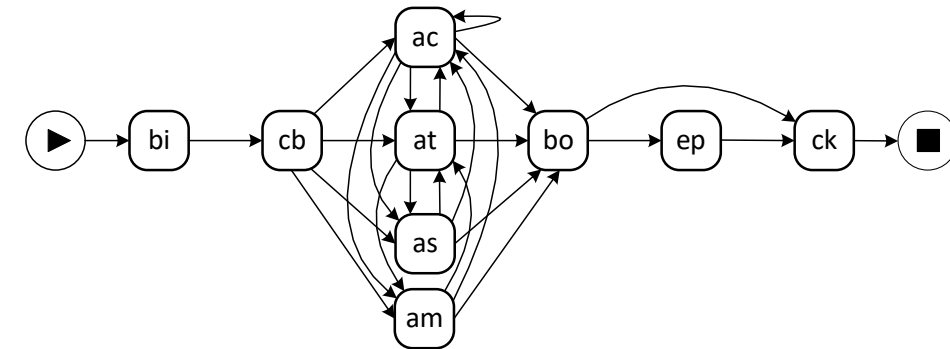


# Event log



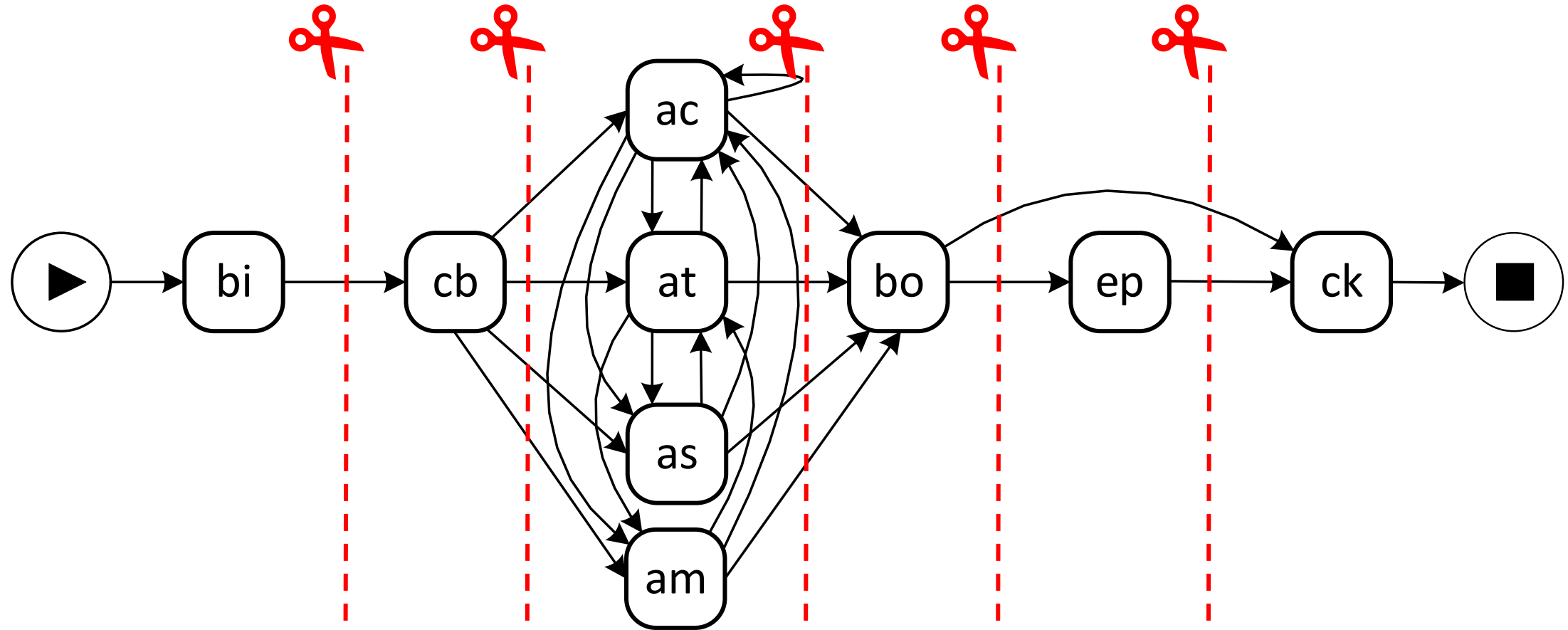
Activities: buy ingredients (bi), create base (cb), add cheese (ac), add tomato (at), add salami (as), add mushrooms (am), bake in oven (bo), eat pizza (ep), and clean kitchen (ck).

# Create a DFG for the whole event log



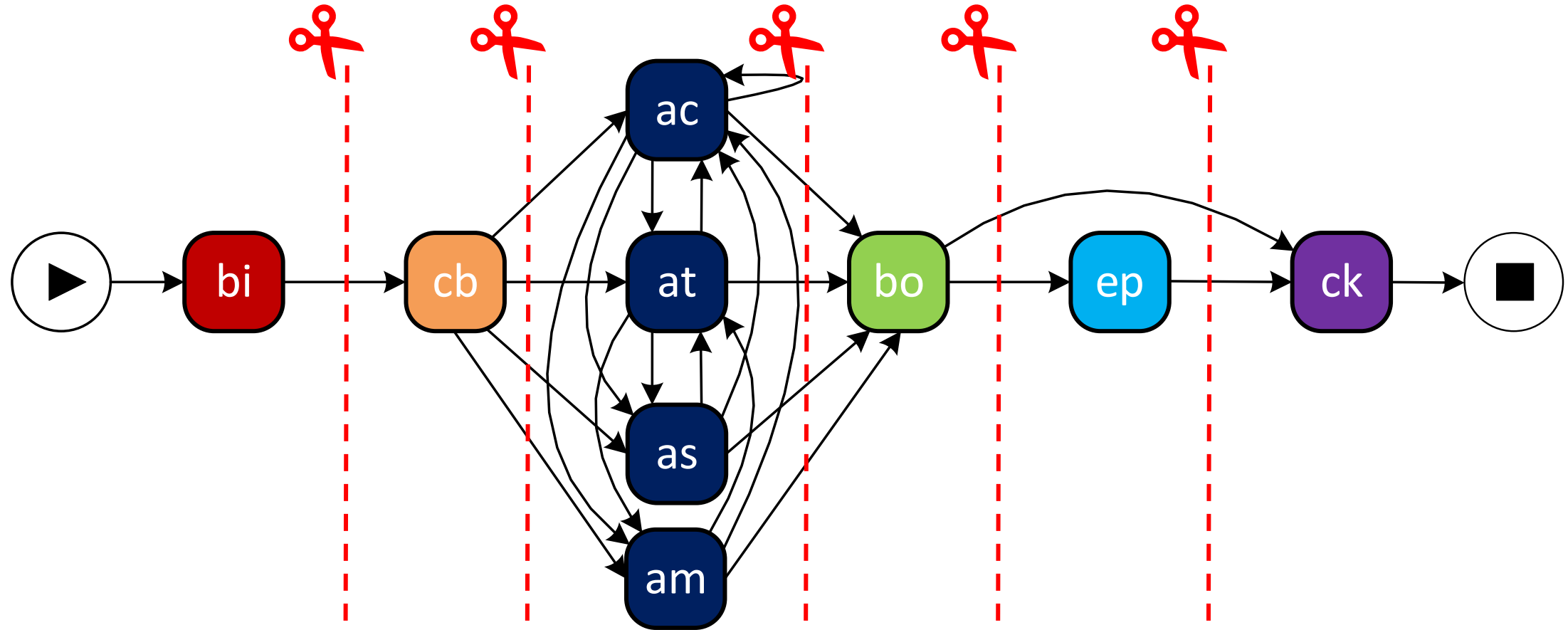
Frequencies omitted for readability

# Apply a sequence cut



There is a sequence cut when the DFG can be split into sequential parts where only “forward connections” are possible. Note that we need to use the non-reflexive transitive closure of  $F$ .

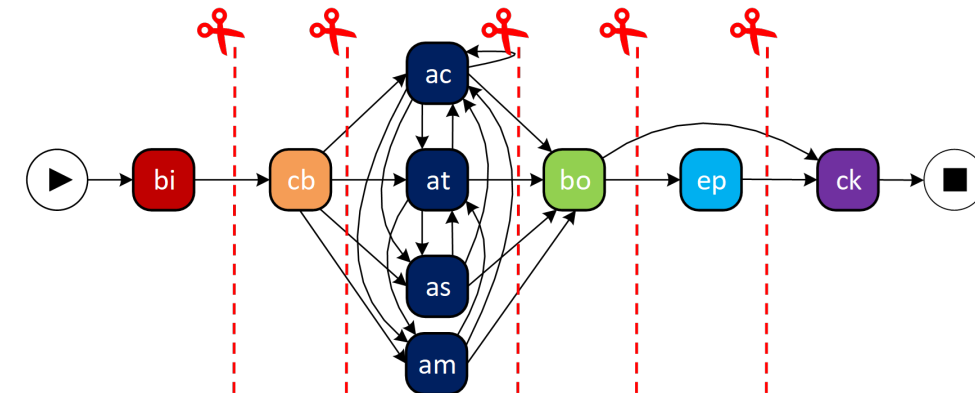
# Sequence cut partitions activities in six subsets



# Color the events based on the partitioning



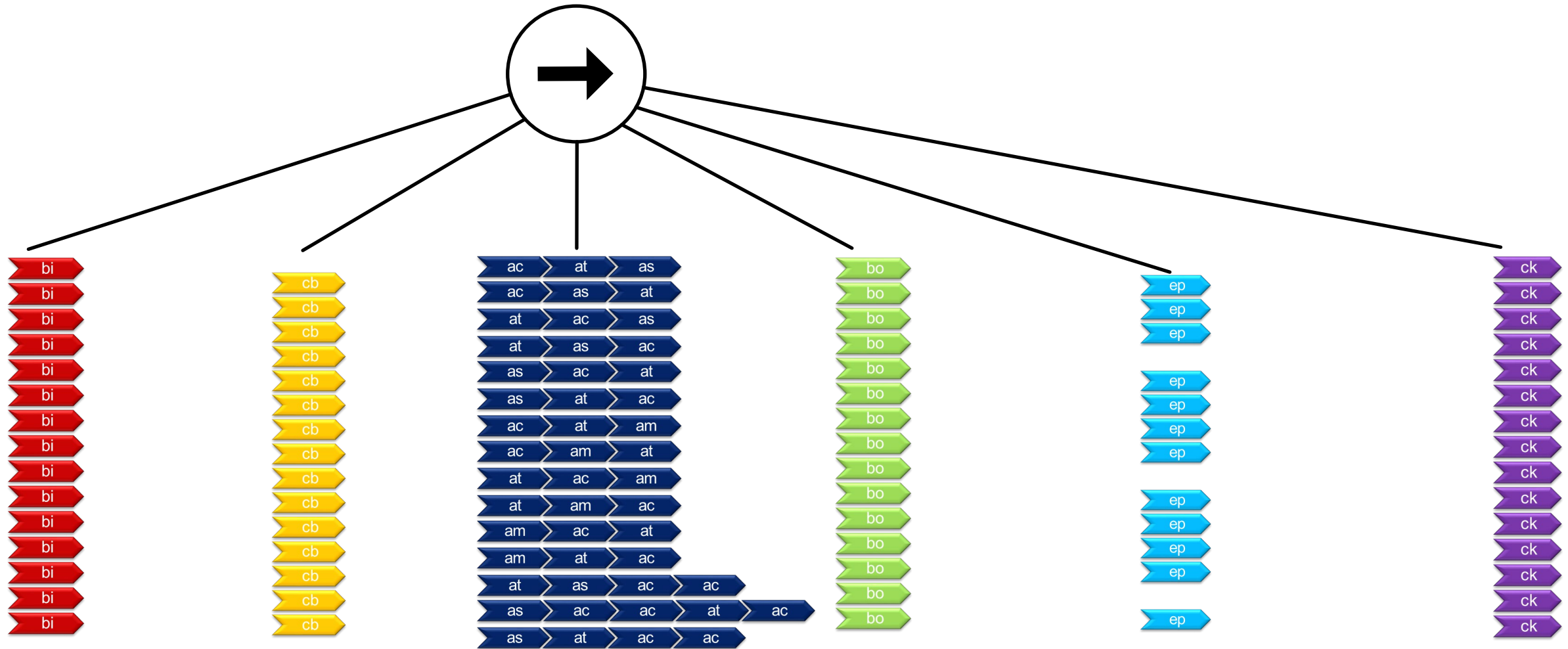
Sequence cut



# Split the event log based on the partitioning

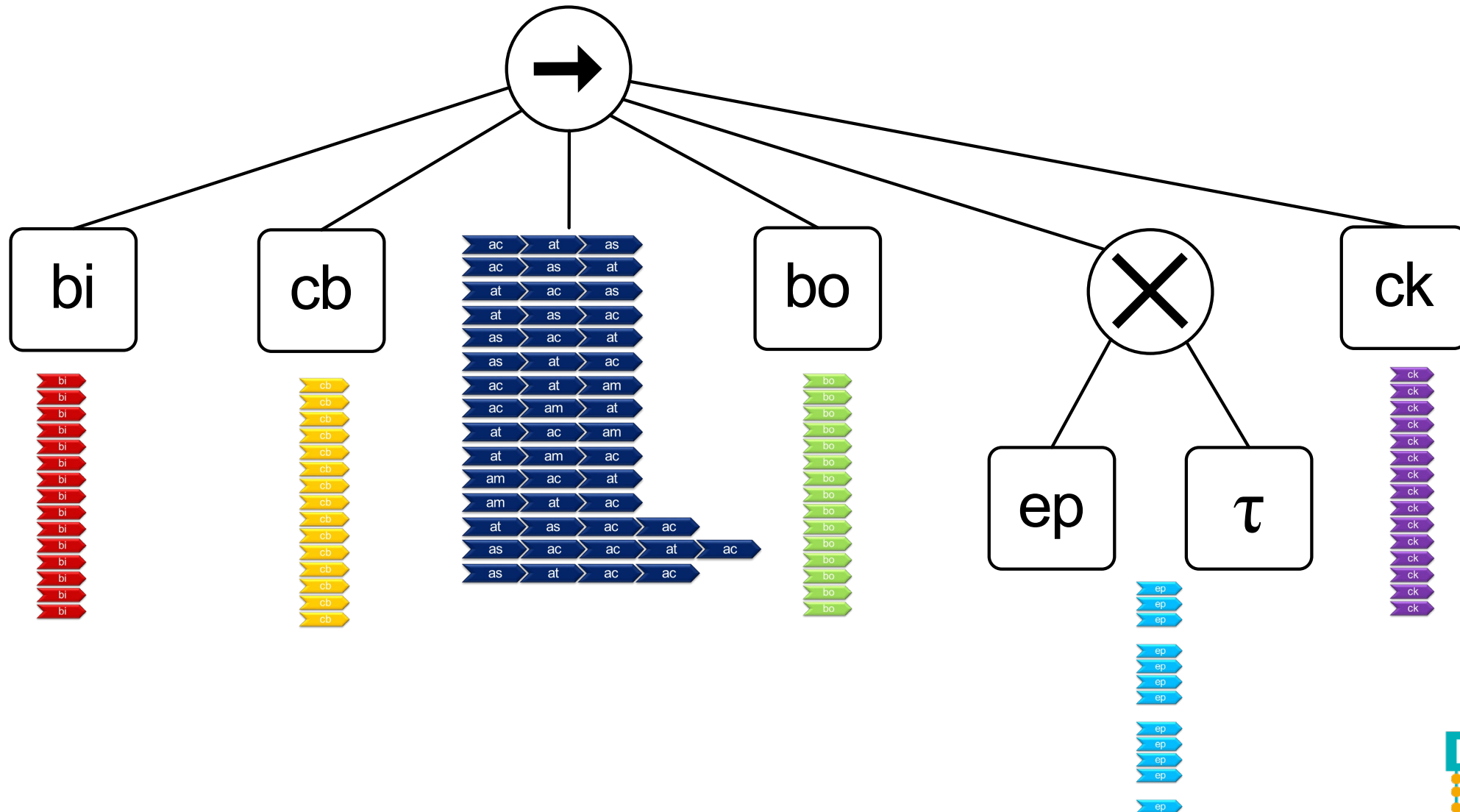


# Recursion: Apply algorithm to all sublogs

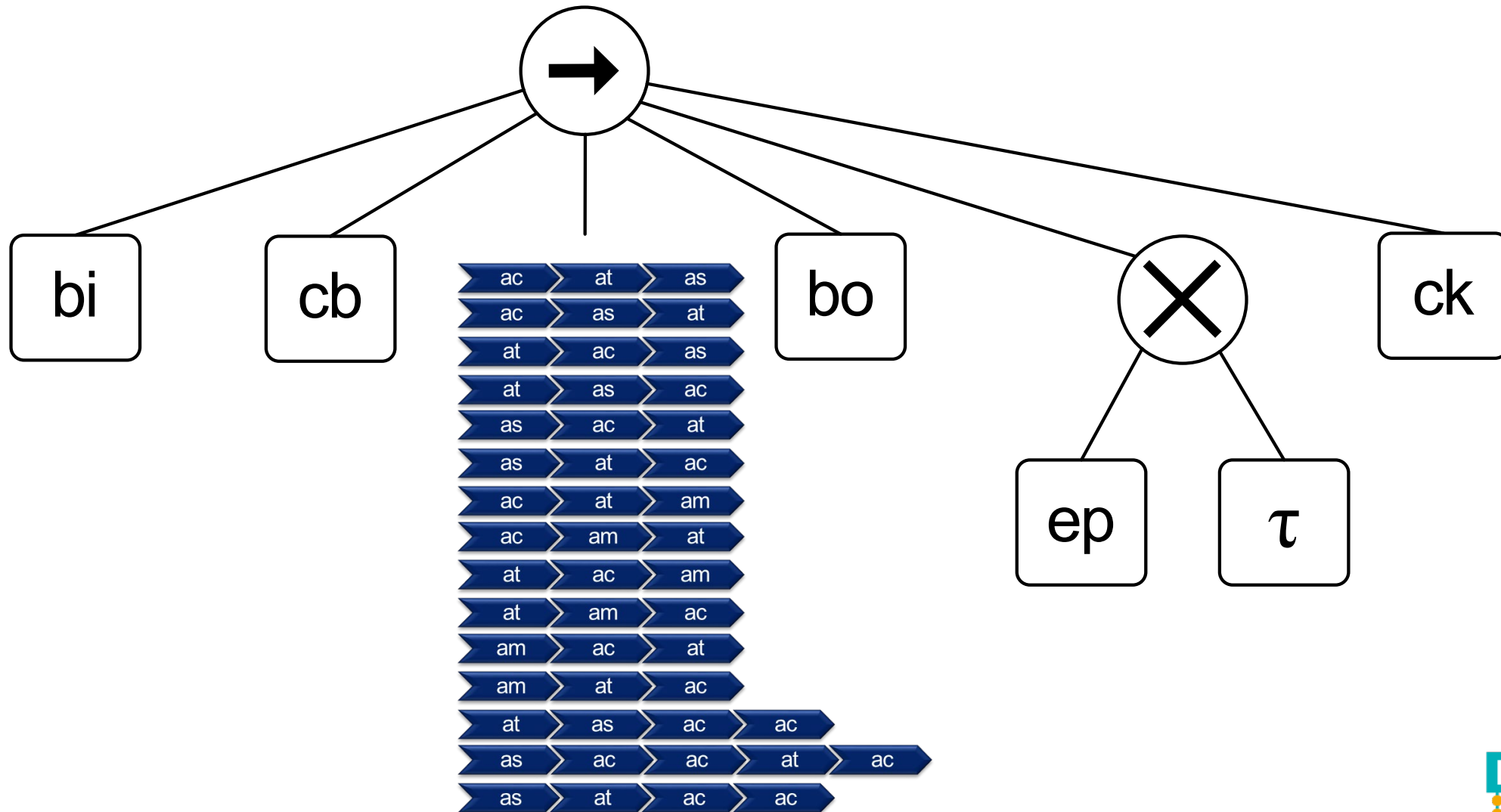


Five of the projected event logs refer to a single activity (base case).

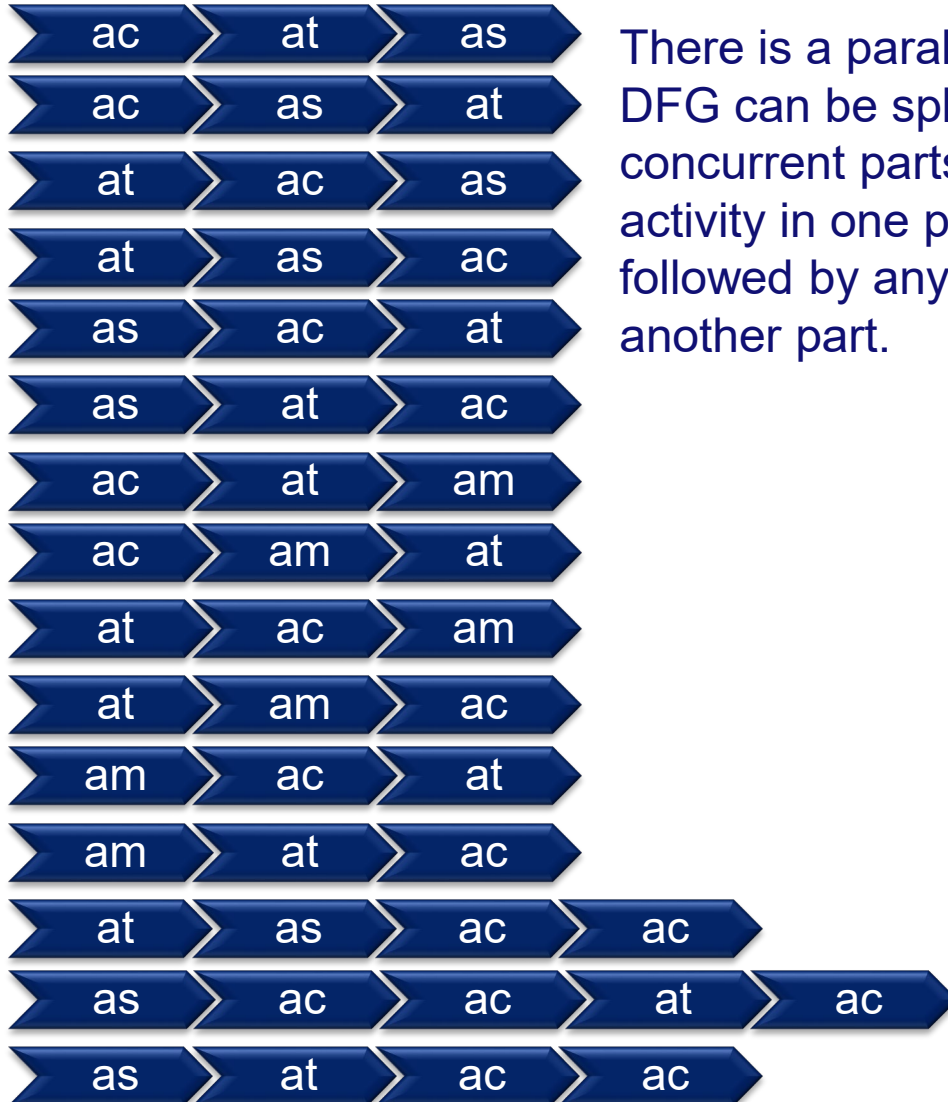
# Handling the base cases (ep can be skipped)



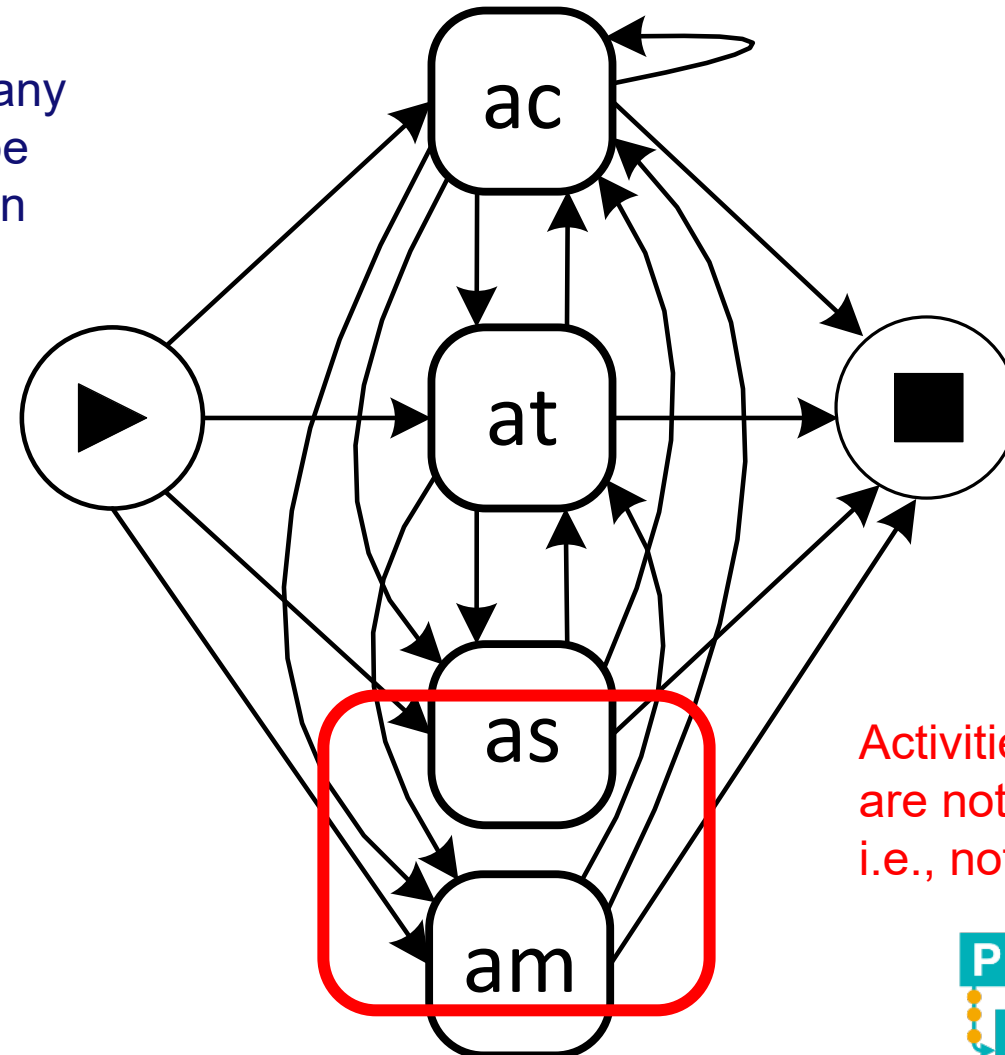
# Only the blue event log remains



# Continue with the blue event log

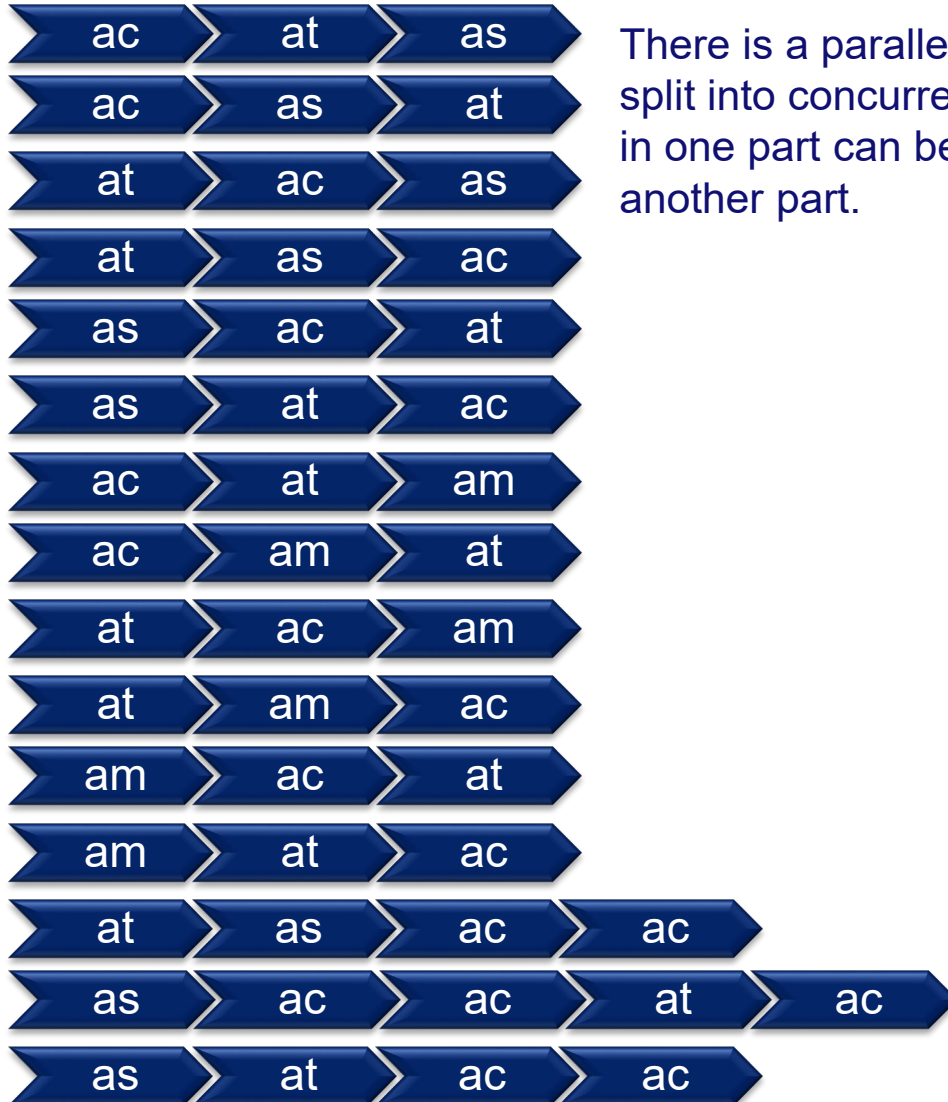


There is a parallel cut when the DFG can be split into concurrent parts where any activity in one part can be followed by any activity in another part.

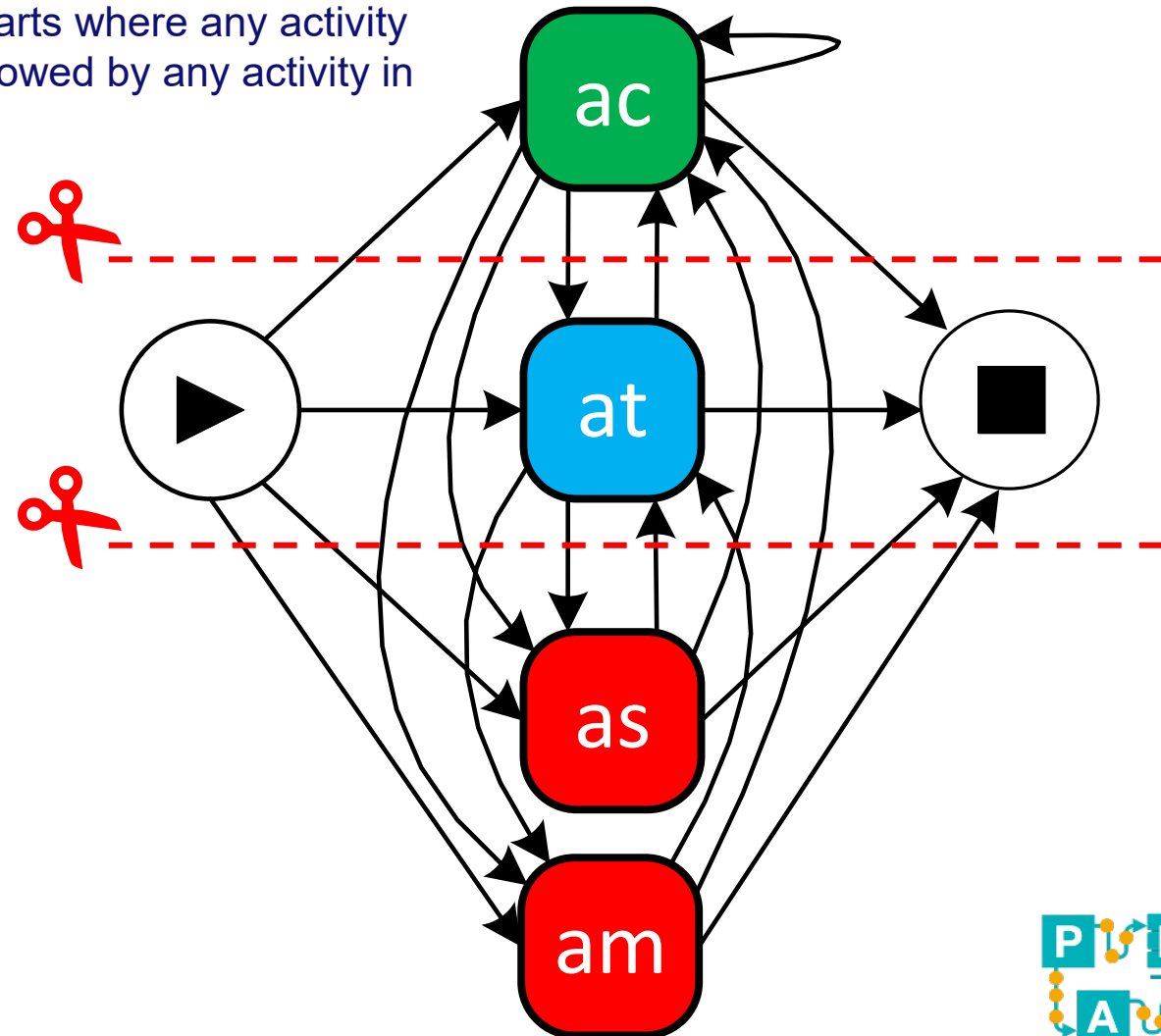


Activities as and am are not connected, i.e., not concurrent

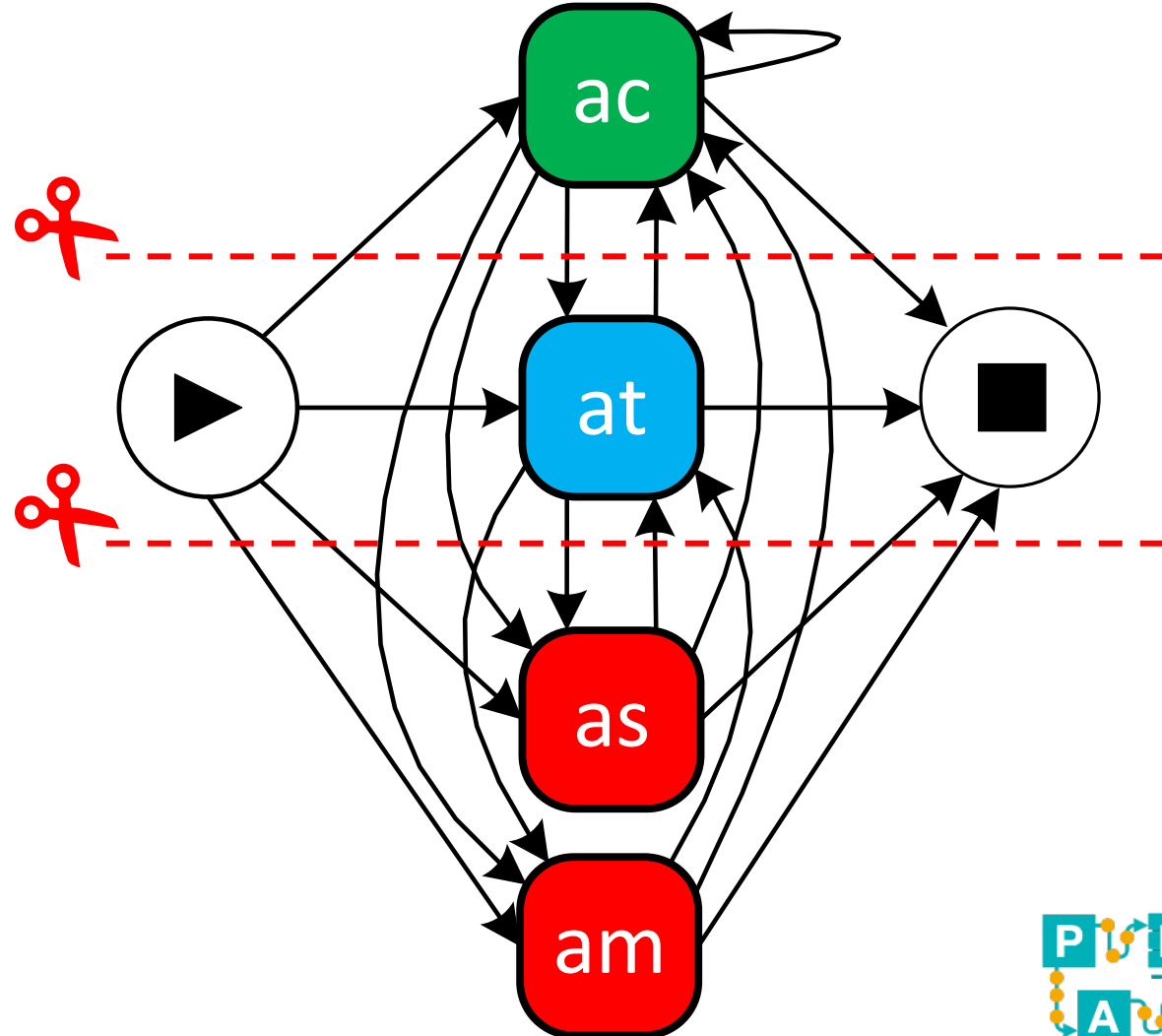
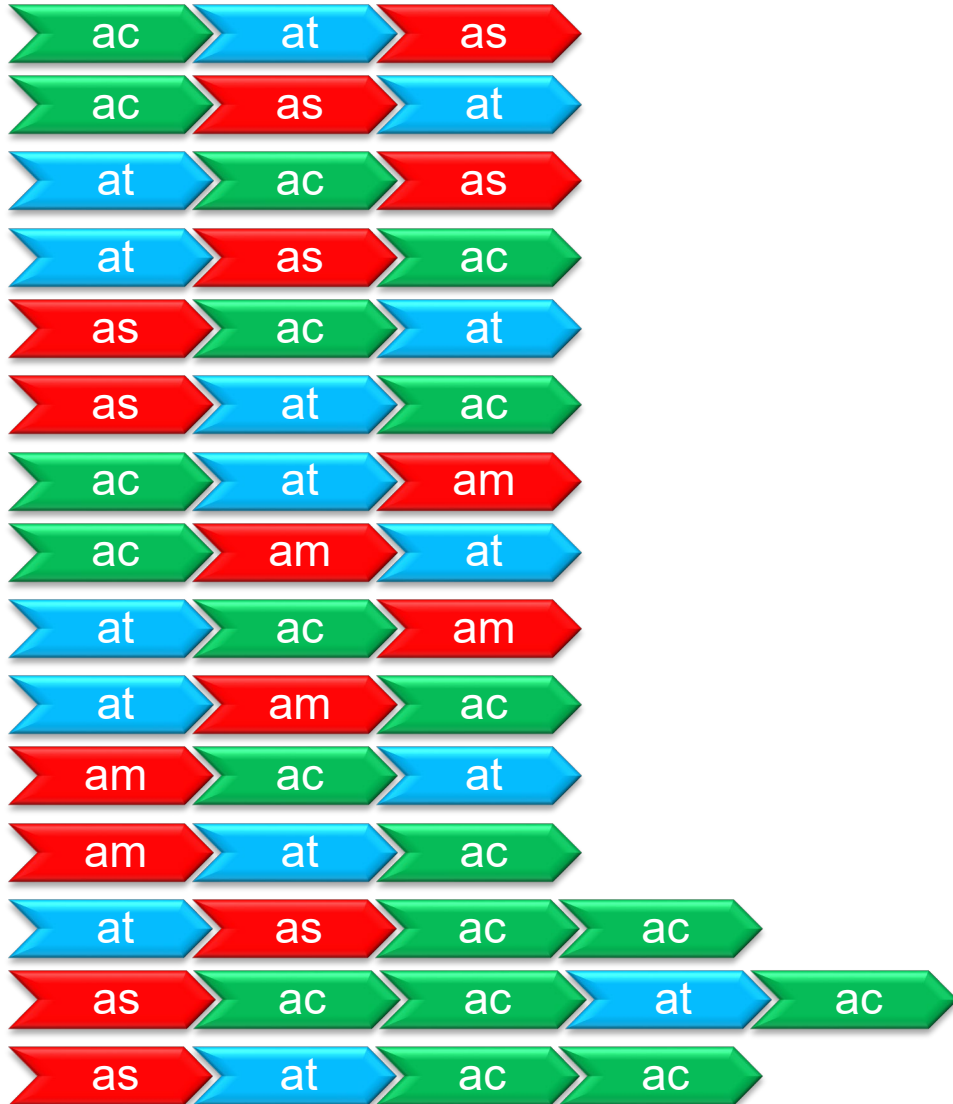
# Apply a parallel cut resulting in three activity groups



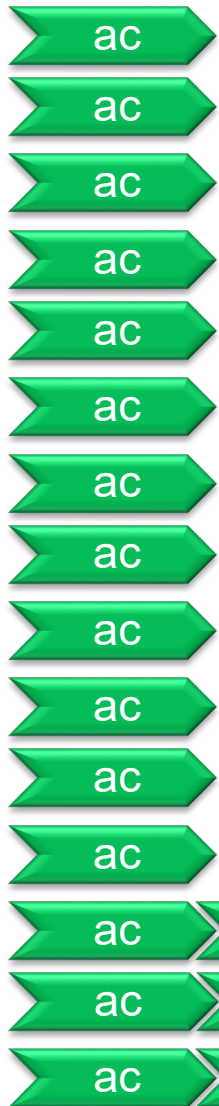
There is a parallel cut when the DFG can be split into concurrent parts where any activity in one part can be followed by any activity in another part.



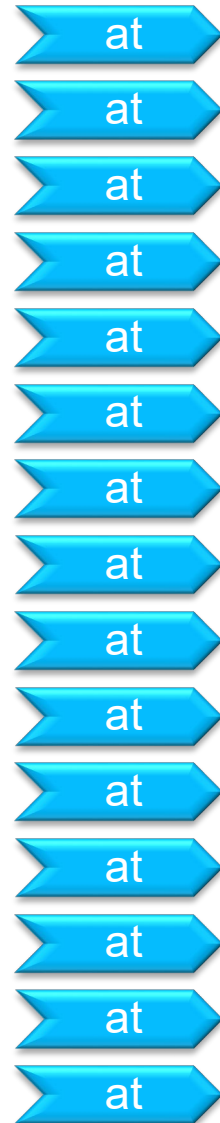
# Apply a parallel cut resulting in three activity groups



# Three new event logs are created



Base case (just  
activity ac)

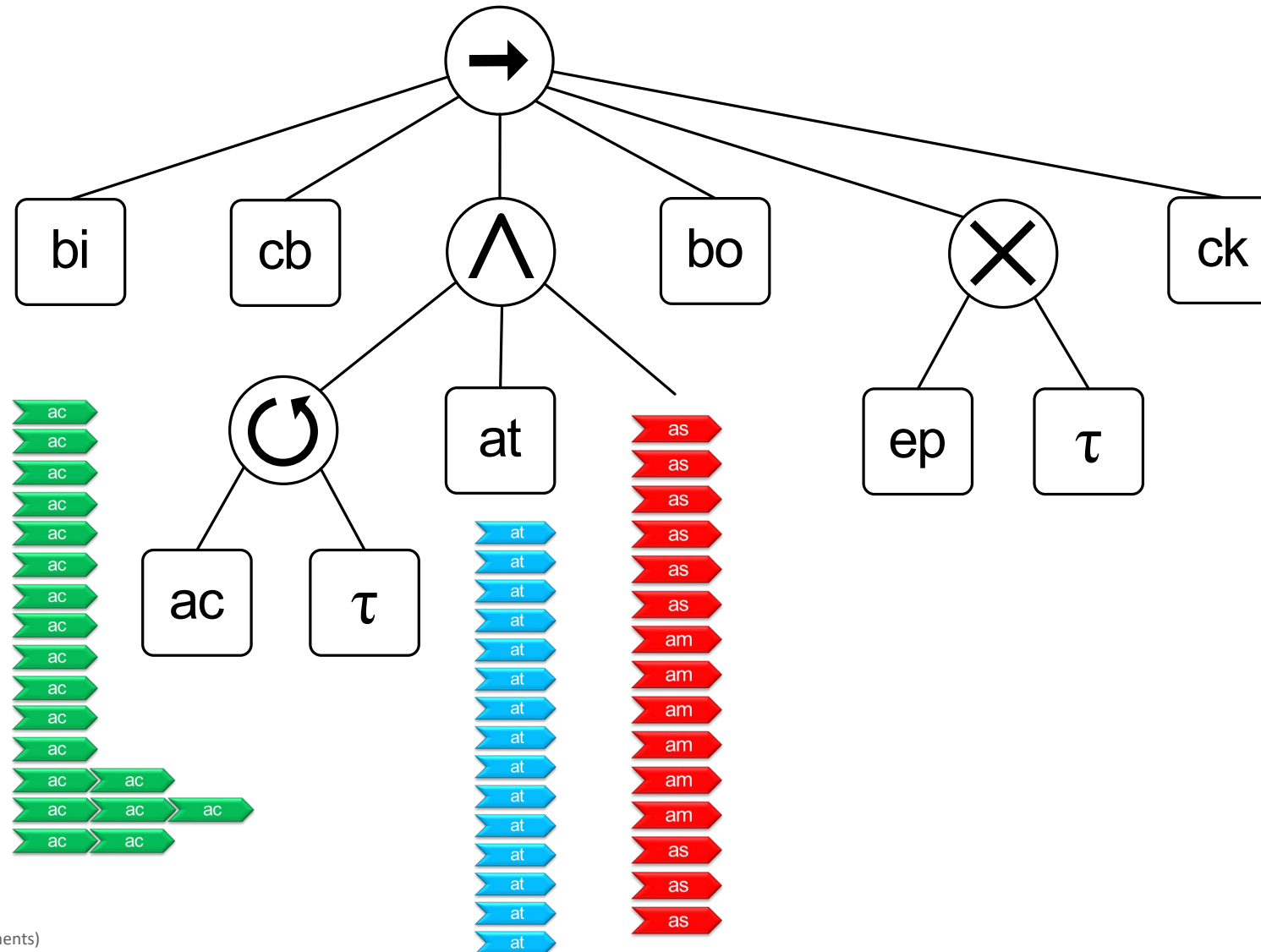


Base case (just  
activity at)

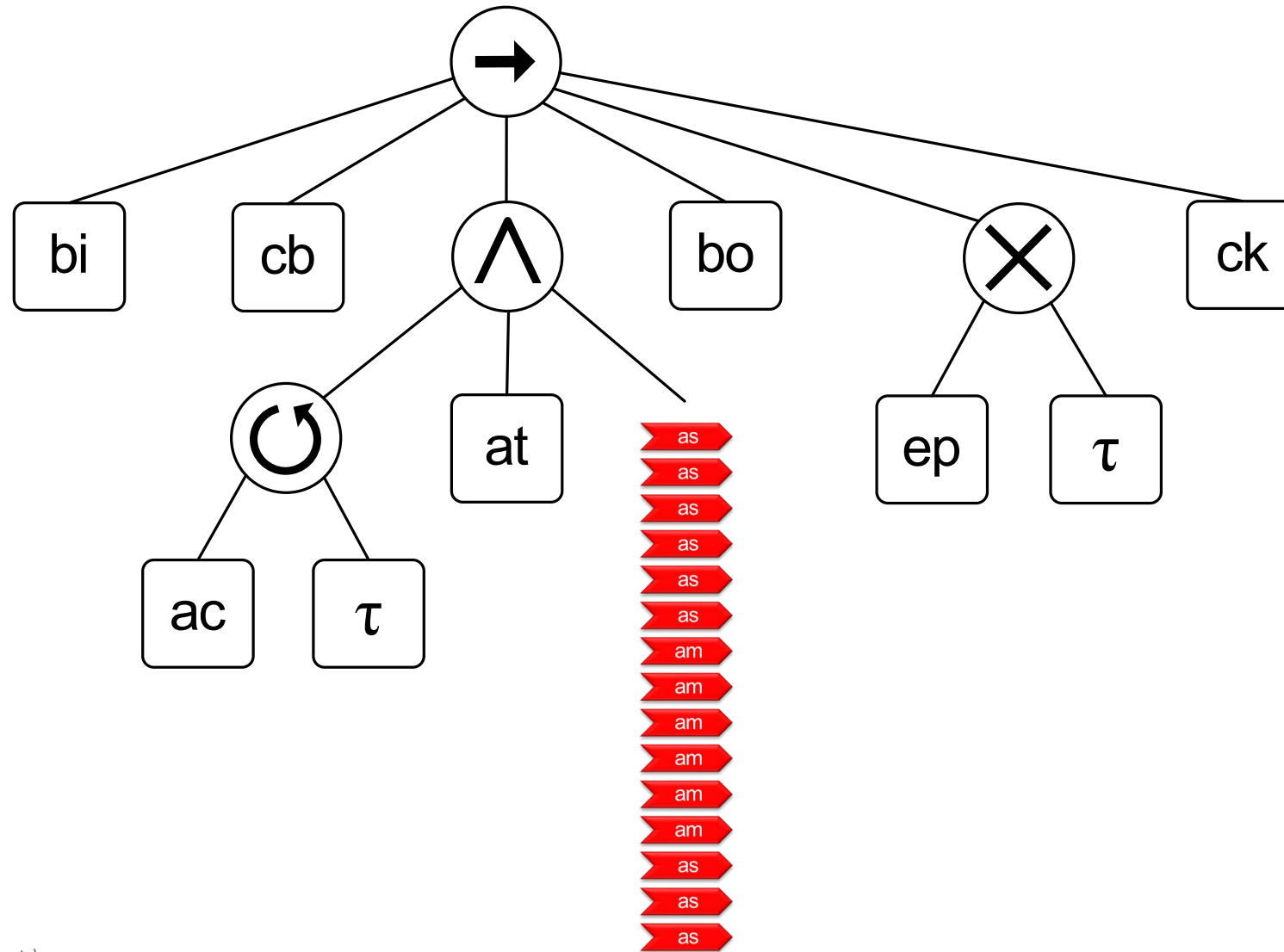


Not a base case, still two  
activities as and am.

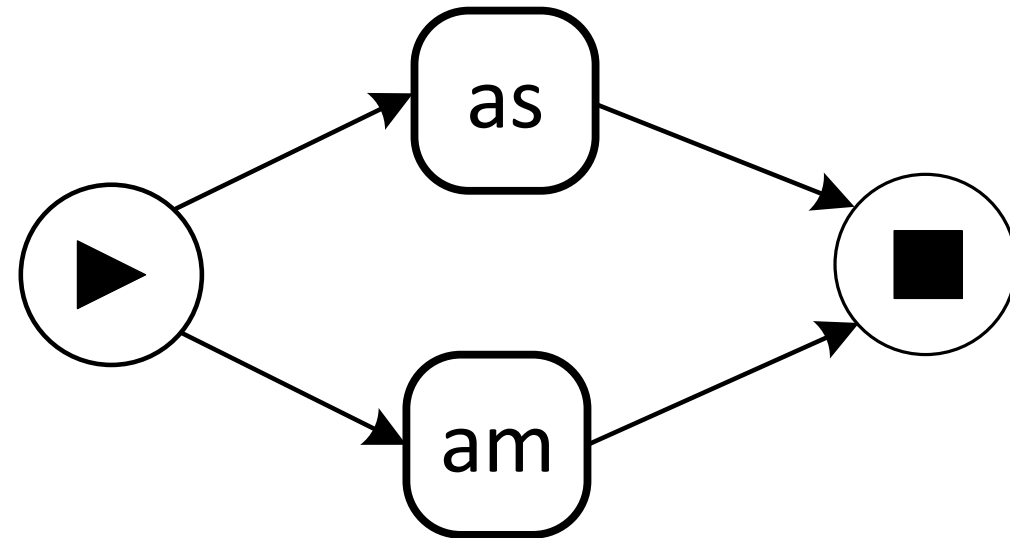
# Handling the base cases (ac can be repeated)



# Only the red event log remains



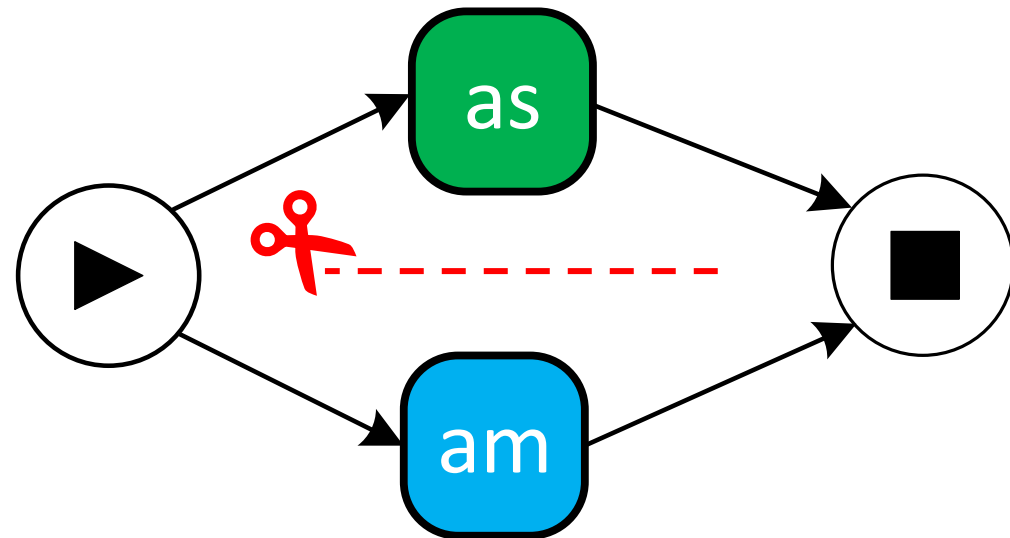
# Continue with the red event log



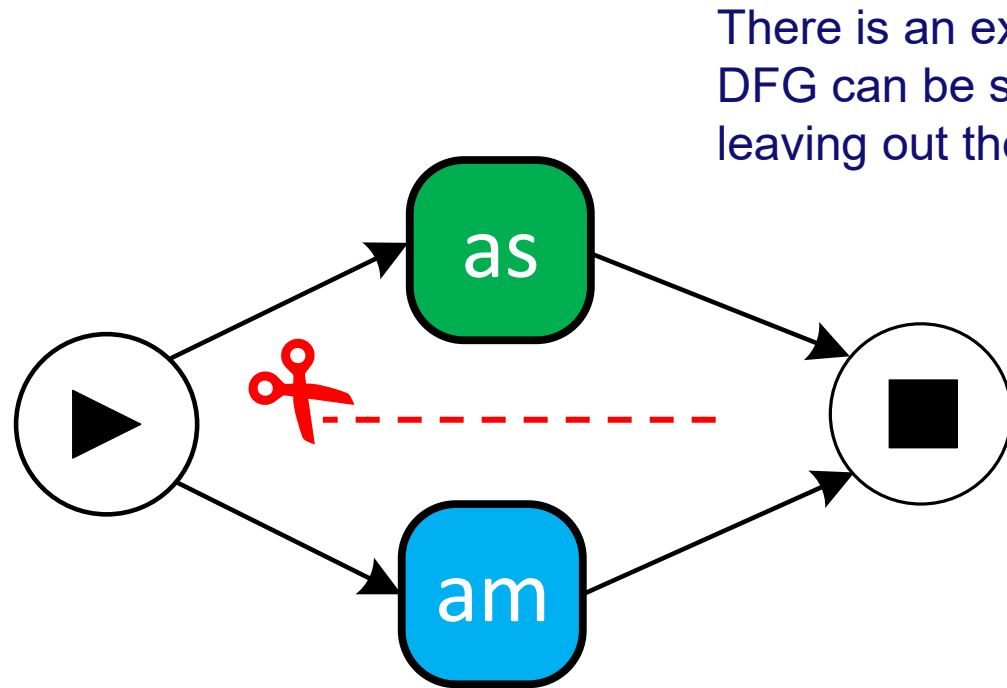
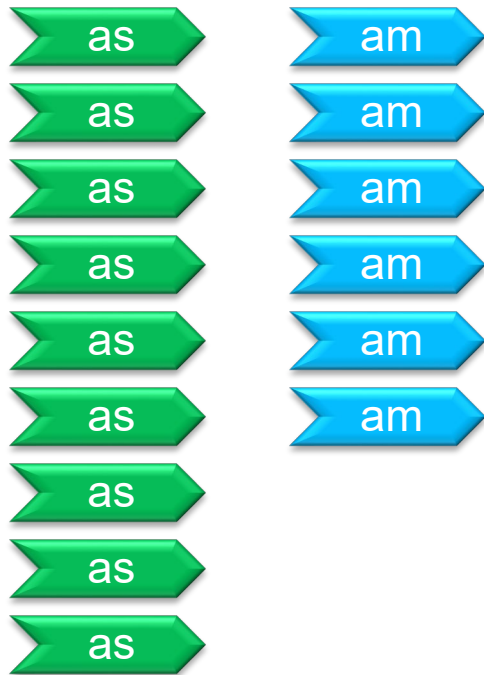
# We find an exclusive-choice cut



There is an exclusive-choice cut when the DFG can be split into disconnected parts after leaving out the artificial start and end.



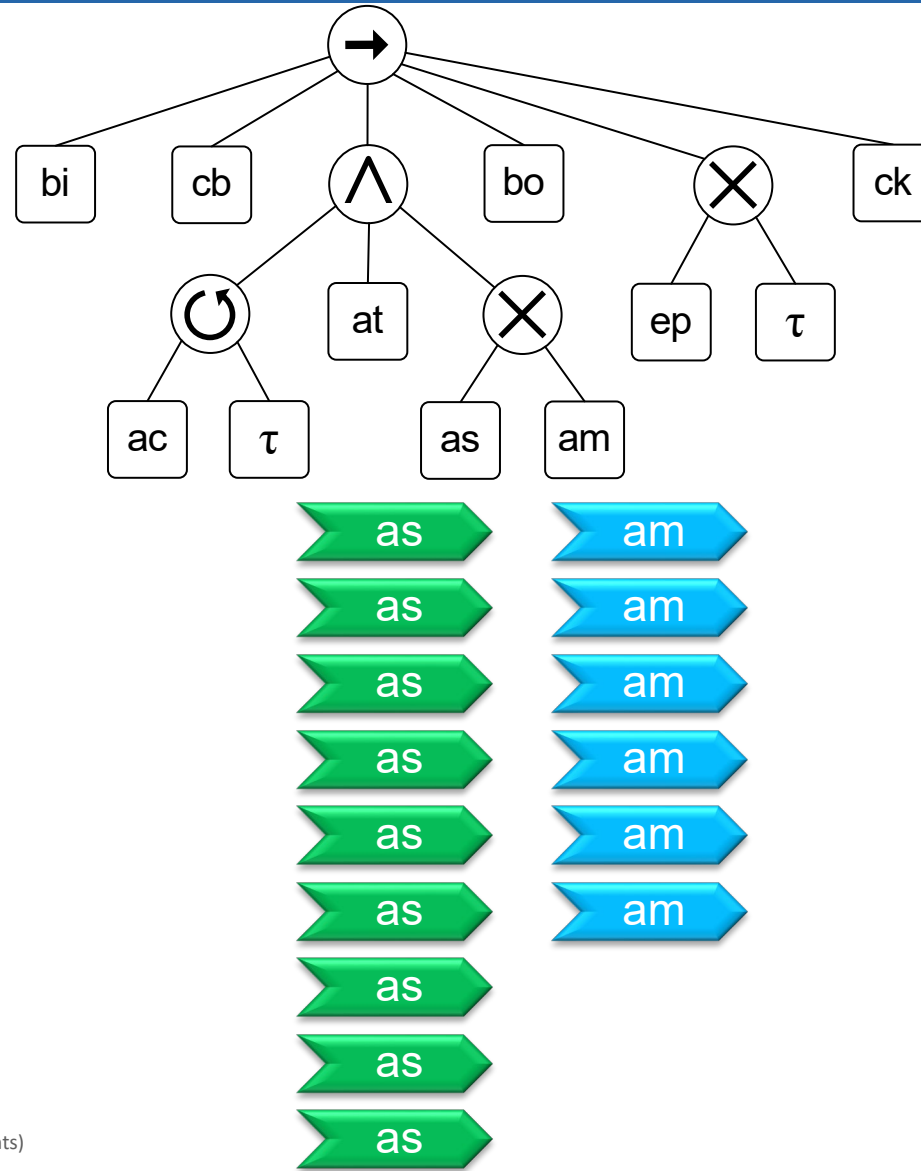
# We find an exclusive-choice cut



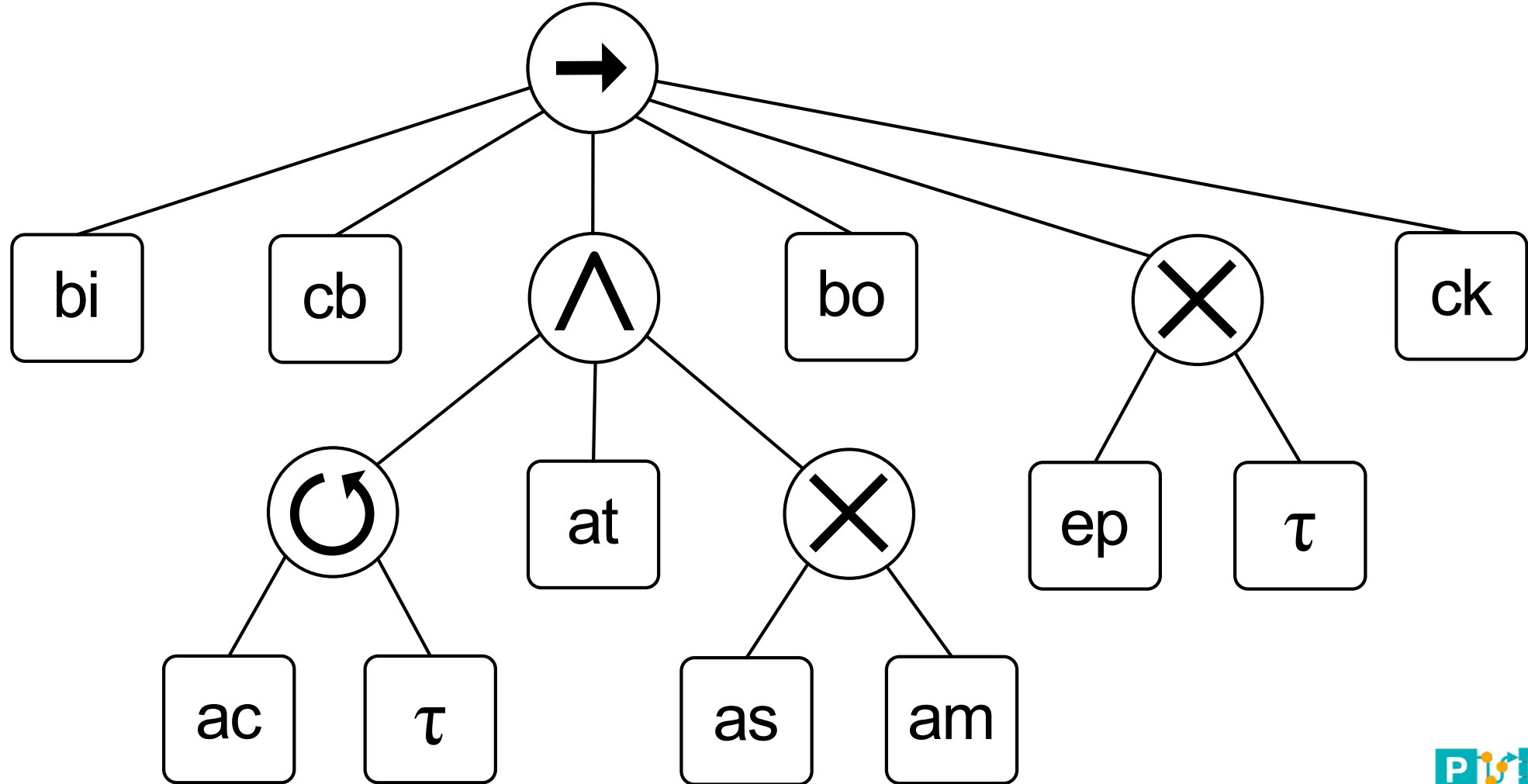
There is an exclusive-choice cut when the DFG can be split into disconnected parts after leaving out the artificial start and end.

Note that projection is now different than for the sequence and parallel cuts.

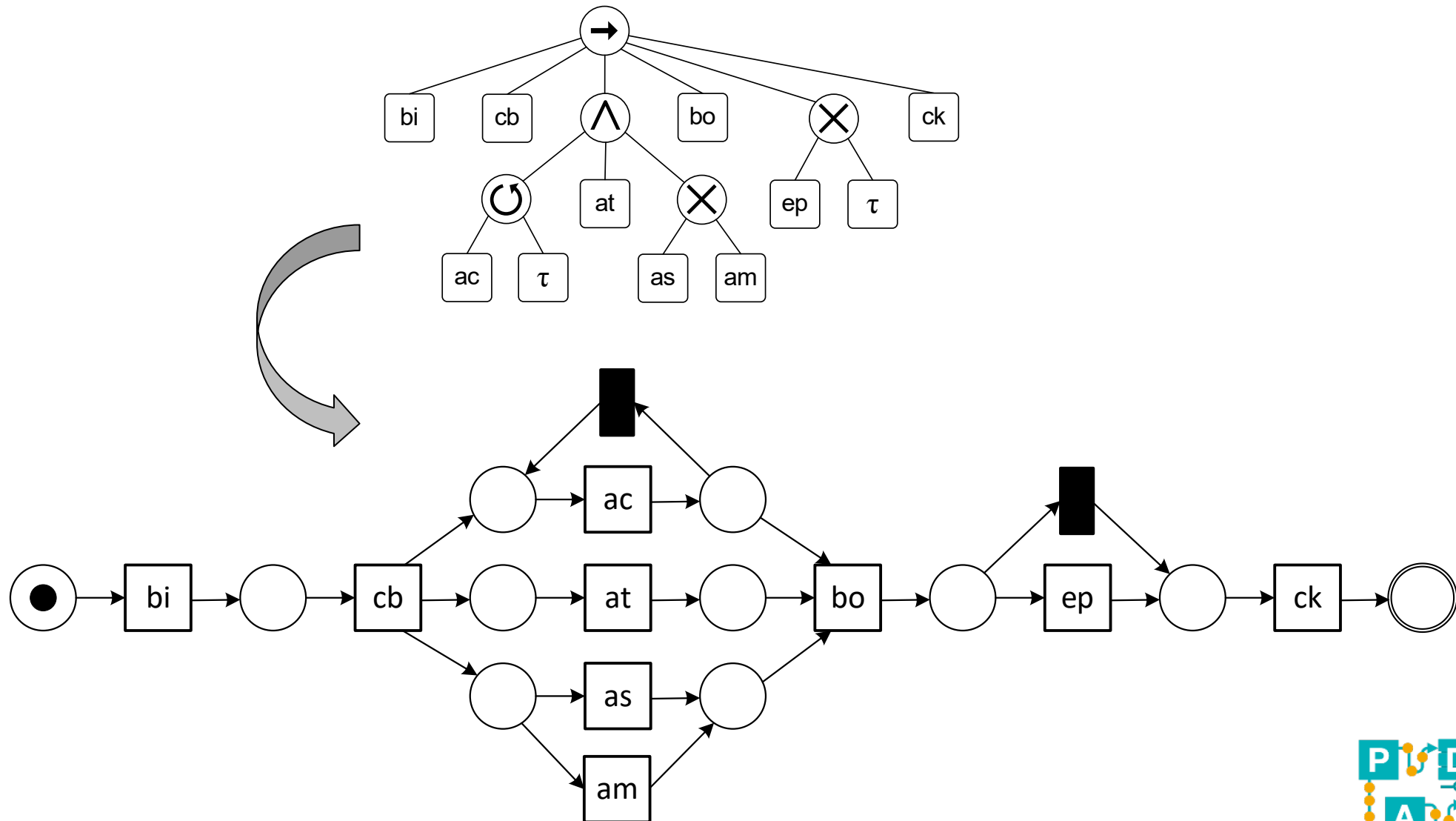
# We end up with two base cases



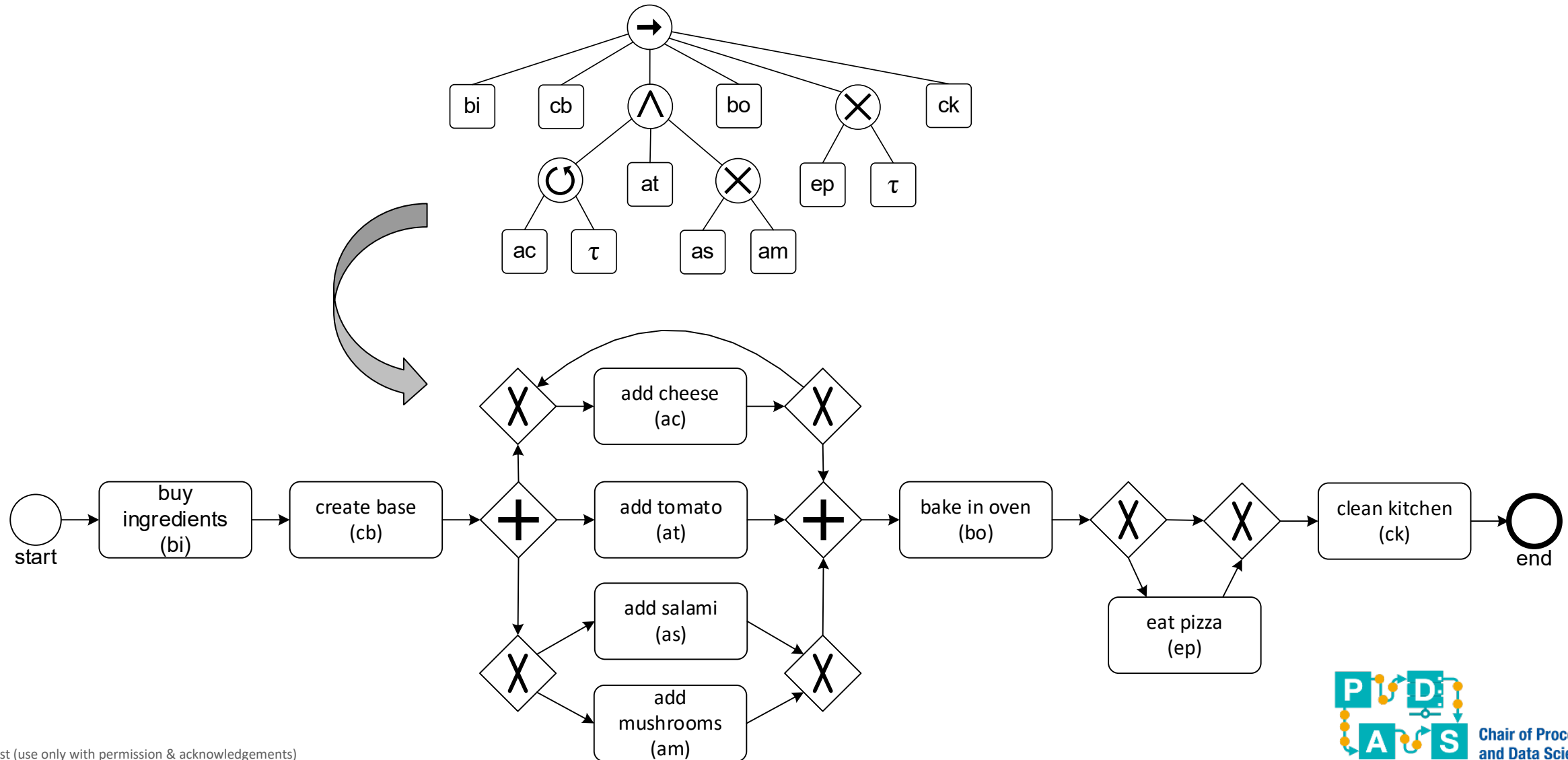
# The process tree returned by the Inductive Mining algorithm



# Can be visualized using Petri nets or BPMN



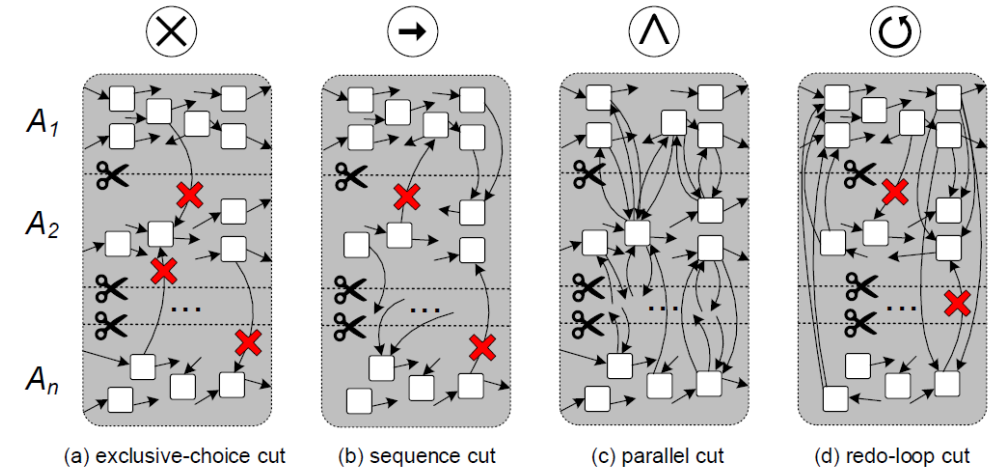
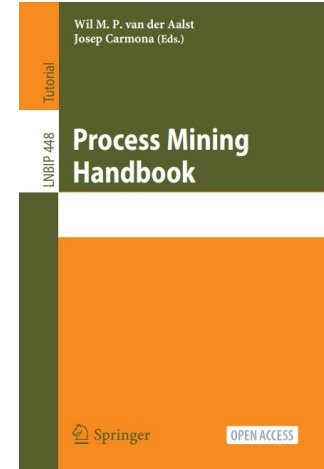
# Can be visualized using Petri nets or BPMN



# Formal definition of the algorithm

**Definition 23 (Sequence, Exclusive-Choice, Parallel, and Redo-Loop Cuts).** Let  $L \in \mathcal{B}(\mathcal{U}_{act}^*)$  be an event log having a DFG  $disc_{DFG}(L) = (A, F)$  based on  $L$  (note that  $A = act(L)$ ) with start activities  $A^{start} = \{a \in A \mid (\blacktriangleright, a) \in F\}$  and end activities  $A^{end} = \{a \in A \mid (a, \blacksquare) \in F\}$ . An  $n$ -ary  $\oplus$ -cut of  $L$  is a partition of  $A$  into  $n \geq 2$  pairwise disjoint subsets  $A_1, A_2, \dots, A_n$  (i.e.,  $A = \bigcup_{i \in \{1, \dots, n\}} A_i$  and  $A_i \cap A_j = \emptyset$  for  $i \neq j$ ) with  $\oplus \in \{\rightarrow, \times, \wedge, \cup\}$ . Such a  $\oplus$ -cut is denoted  $(\oplus, A_1, A_2, \dots, A_n)$ . For each type of operator  $\oplus \in \{\rightarrow, \times, \wedge, \cup\}$  specific conditions apply:

- An exclusive-choice cut of  $L$  is a cut  $(\times, A_1, A_2, \dots, A_n)$  such that
  - $\forall_{i,j \in \{1, \dots, n\}} \forall_{a \in A_i} \forall_{b \in A_j} i \neq j \Rightarrow (a, b) \notin F$ .
- A sequence cut of  $L$  is a cut  $(\rightarrow, A_1, A_2, \dots, A_n)$  such that
  - $\forall_{i,j \in \{1, \dots, n\}} \forall_{a \in A_i} \forall_{b \in A_j} i < j \Rightarrow ((a, b) \in F^+ \wedge (b, a) \notin F^+)$ .  
(Note that  $F^+$  is the non-reflexive transitive closure of  $F$ , i.e.,  $(a, b) \in F^+$  means that there is a path from  $a$  to  $b$  in the DFG.)
- A parallel cut of  $L$  is a cut  $(\wedge, A_1, A_2, \dots, A_n)$  such that
  - $\forall_{i \in \{1, \dots, n\}} A_i \cap A^{start} \neq \emptyset \wedge A_i \cap A^{end} \neq \emptyset$  and
  - $\forall_{i,j \in \{1, \dots, n\}} \forall_{a \in A_i} \forall_{b \in A_j} i \neq j \Rightarrow (a, b) \in F$ .
- A redo-loop cut of  $L$  is a cut  $(\cup, A_1, A_2, \dots, A_n)$  such that
  - $A^{start} \cup A^{end} \subseteq A_1$ ,
  - $\forall_{i,j \in \{2, \dots, n\}} \forall_{a \in A_i} \forall_{b \in A_j} i \neq j \Rightarrow (a, b) \notin F$ ,
  - $\{a \in A_1 \mid (a, b) \in F \wedge b \notin A_1\} = A^{end}$ ,
  - $\{a \in A_1 \mid (b, a) \in F \wedge b \notin A_1\} = A^{start}$ ,
  - $\forall_{(a,b) \in F} a \in A_1 \wedge b \notin A_1 \Rightarrow \forall_{a' \in A^{end}} (a', b) \in F$ , and
  - $\forall_{(b,a) \in F} a \in A_1 \wedge b \notin A_1 \Rightarrow \forall_{a' \in A^{start}} (b, a') \in F$ .



# Strong theoretical guarantees

- **Guaranteed to be able to replay the whole event log, i.e., perfect recall (if desired).**
- **“Rediscoverability” of process trees with unique activities (if the log is directly-follows complete).**
- **Scalable enough for real-life events logs (more scalable variants available) and able to filter out infrequent behavior.**
- **Implemented in open source tools like ProM and PM4py, and several commercial tools, including Celonis.**



# Question: Is this machine learning?

# Optimization for Process Discovery

# Example: ILP Mining

**Definition (Basic ILP formulation)** Let  $L \subseteq T^*$  be an event log (cardinalities are not considered).  $A$  and  $A'$  are the matrices defined for the language of  $L$ .  $ILP_L$  is the integer programming problem:

<b>Minimize</b>	$c + \mathbf{1}^T \cdot (c \cdot \mathbf{1} + A \cdot (\mathbf{x} - \mathbf{y}))$	minimal and “interesting” regions
<b>such that</b>	$c \cdot \mathbf{1} + A' \cdot \mathbf{x} - A \cdot \mathbf{y} \geq \mathbf{0}$	$(\mathbf{x}, \mathbf{y}, c)$ is a region
	$\mathbf{1}^T \cdot \mathbf{x} + \mathbf{1}^T \cdot \mathbf{y} \geq 1$	there is at least one edge
	$\mathbf{0} \leq \mathbf{x} \leq \mathbf{1}$	$\mathbf{x} \in \{0, 1\}^T$
	$\mathbf{0} \leq \mathbf{y} \leq \mathbf{1}$	$\mathbf{y} \in \{0, 1\}^T$
	$0 \leq c \leq 1$	$c \in \{0, 1\}$

Joint work with Boudewijn van Dongen, Cor Hurkens, Bas van Zelst, et al.

Finding places in a Petri net

# Example Integer Linear Program (ILP) to discover places in a Petri net

$$L = \{\langle a, c, d \rangle, \langle b, c, e \rangle\}$$

Minimize  $c + (1, 1, 1, 1, 1, 1) \cdot (c \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_a - y_a \\ x_b - y_b \\ x_c - y_c \\ x_d - y_d \\ x_e - y_e \end{pmatrix})$

such that:

$$c \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_a \\ x_b \\ x_c \\ x_d \\ x_e \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_a \\ y_b \\ y_c \\ y_d \\ y_e \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

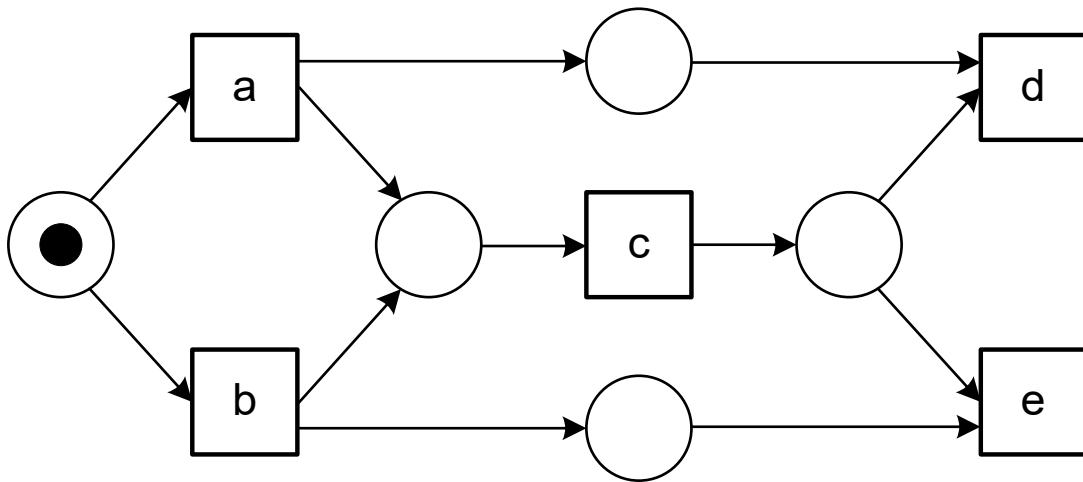
$$(1, 1, 1, 1, 1) \cdot \begin{pmatrix} x_a \\ x_b \\ x_c \\ x_d \\ x_e \end{pmatrix} + (1, 1, 1, 1, 1) \cdot \begin{pmatrix} y_a \\ y_b \\ y_c \\ y_d \\ y_e \end{pmatrix} \geq 1$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \leq \begin{pmatrix} x_a \\ x_b \\ x_c \\ x_d \end{pmatrix} \leq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \leq \begin{pmatrix} y_a \\ y_b \\ y_c \\ y_d \end{pmatrix} \leq \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

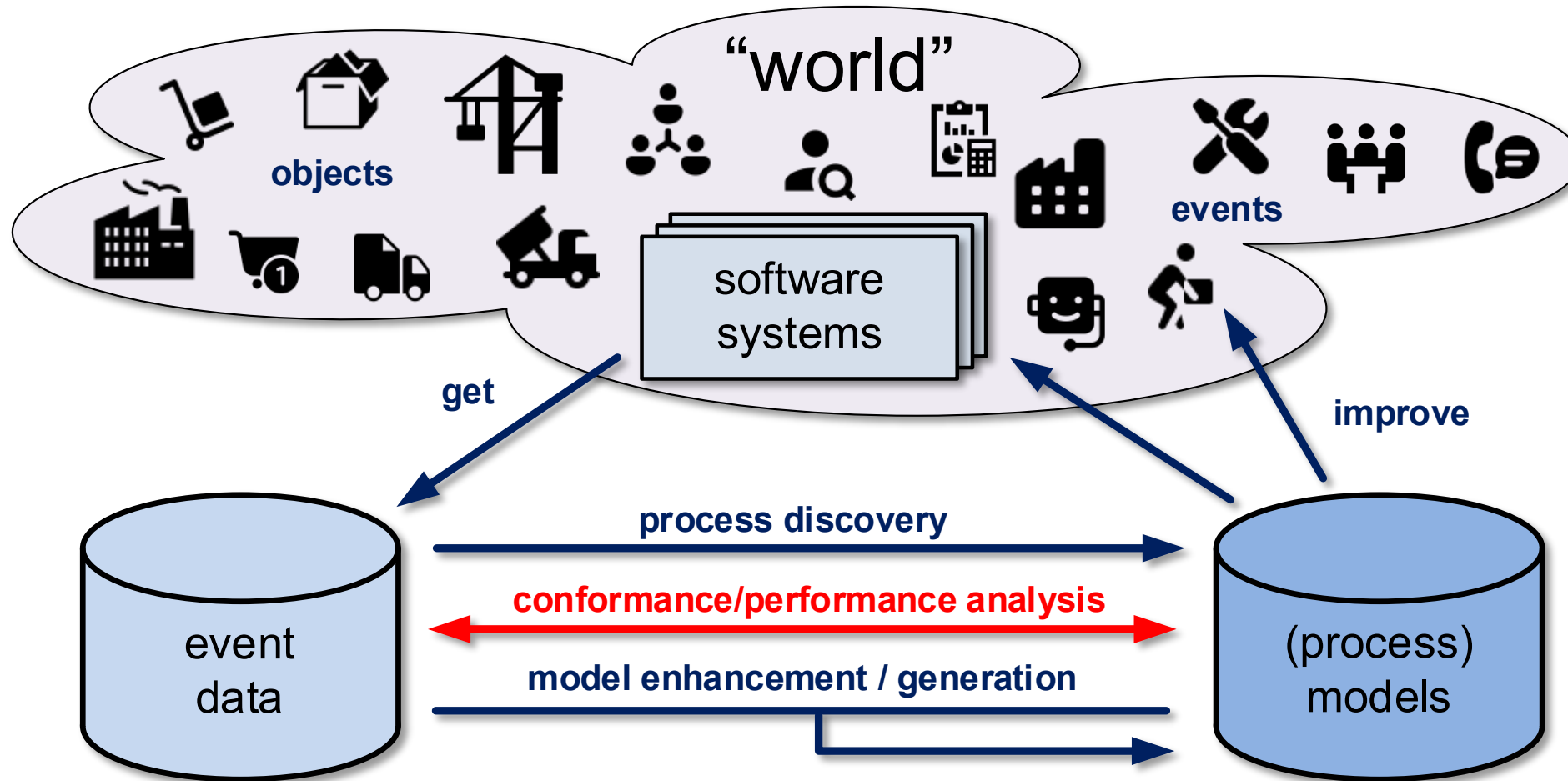
each place is a solution  
(skipping some details, e.g., empty at the end)

$$0 \leq c \leq 1$$



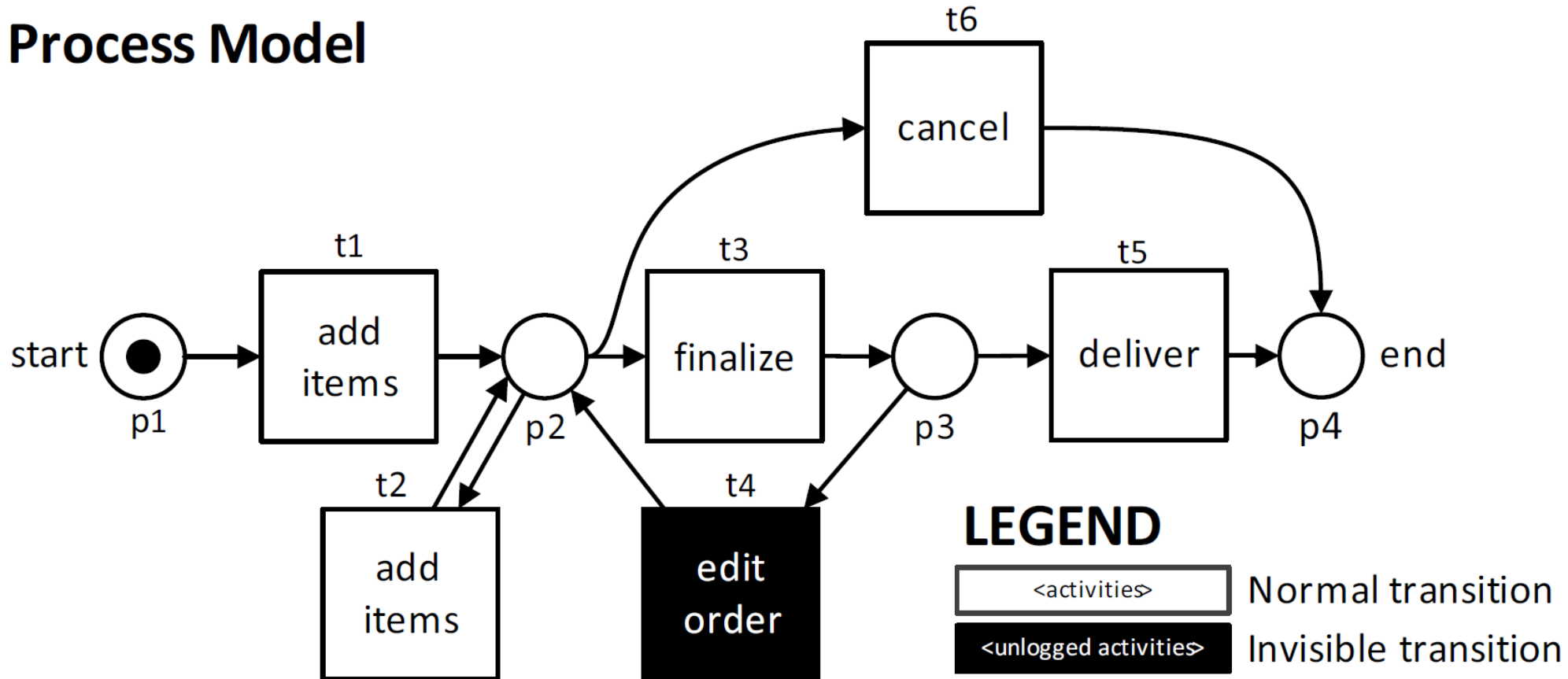
# Optimization for Conformance Checking

# Conformance Checking



# Computing alignments

## Process Model

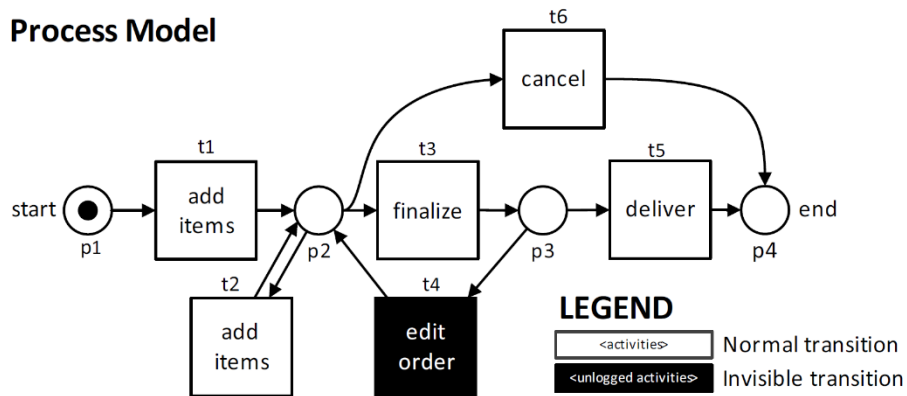


**Trace :** *<add items, add items, finalize, finalize>*

Joint work with Arya Adriansyah and Boudewijn van Dongen

# Two example alignments

## Process Model



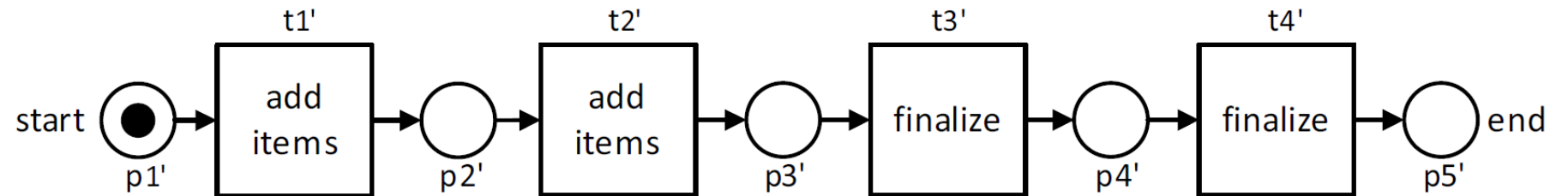
**Trace :** <add items, add items, finalize, finalize>

$$\gamma_1 = \begin{array}{|c|c|c|c|c|} \hline \text{add items} & \text{add items} & \text{finalize} & \text{finalize} & \gg \\ \hline t_1 & t_2 & t_3 & & t_5 \\ \hline \end{array}$$

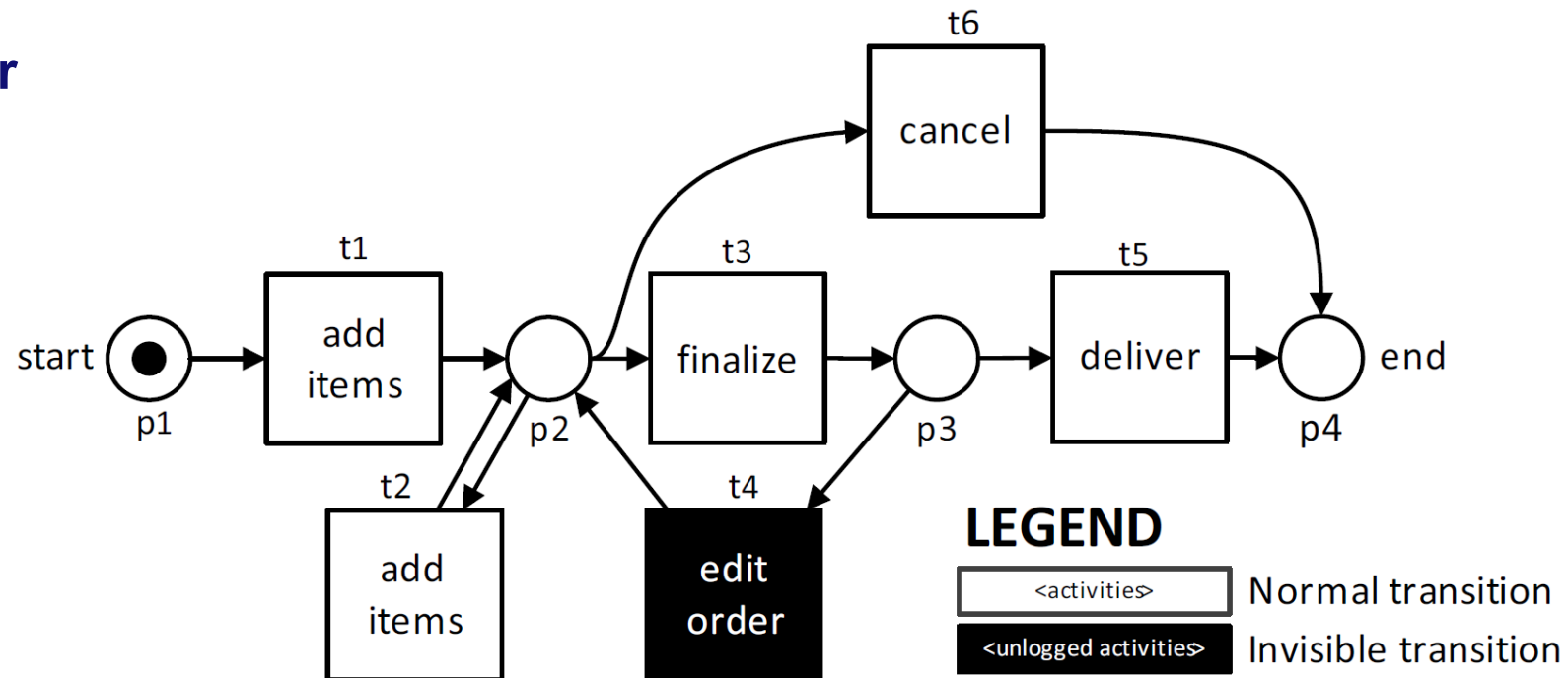
$$\gamma_2 = \begin{array}{|c|c|c|c|c|c|} \hline \text{add items} & \text{add items} & \text{finalize} & \gg & \text{finalize} & \gg \\ \hline t_1 & t_2 & t_3 & t_4 & t_3 & t_5 \\ \hline \end{array}$$

# Two Petri nets

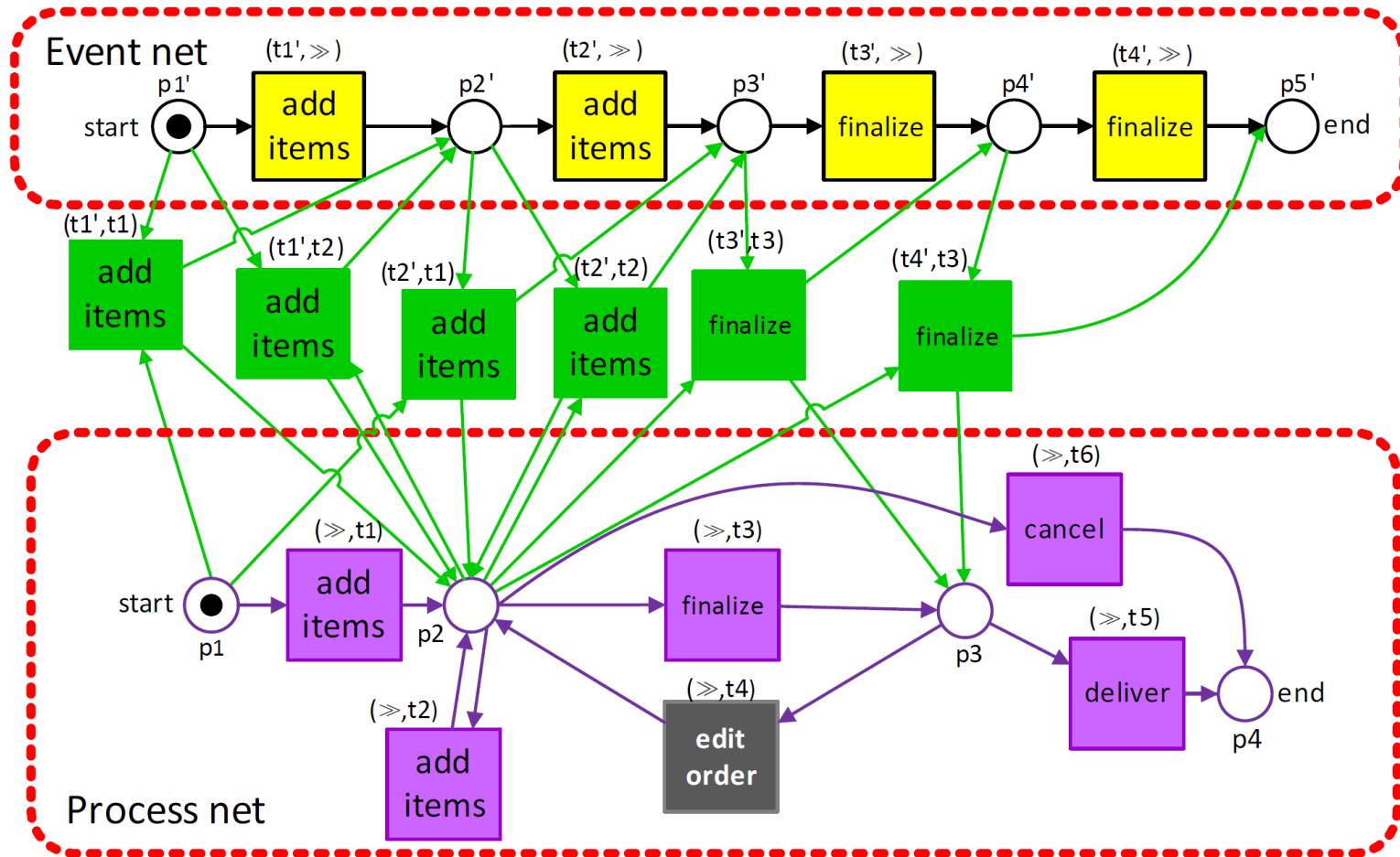
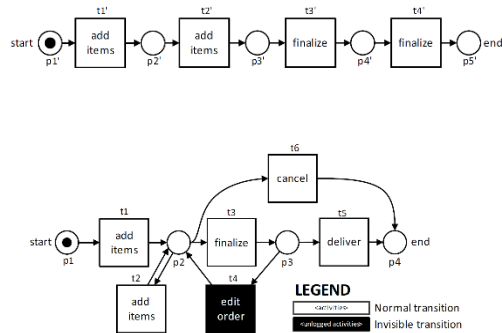
## Observed behavior



## Modeled behavior



# Product net of the process model and the trace model



## LEGEND

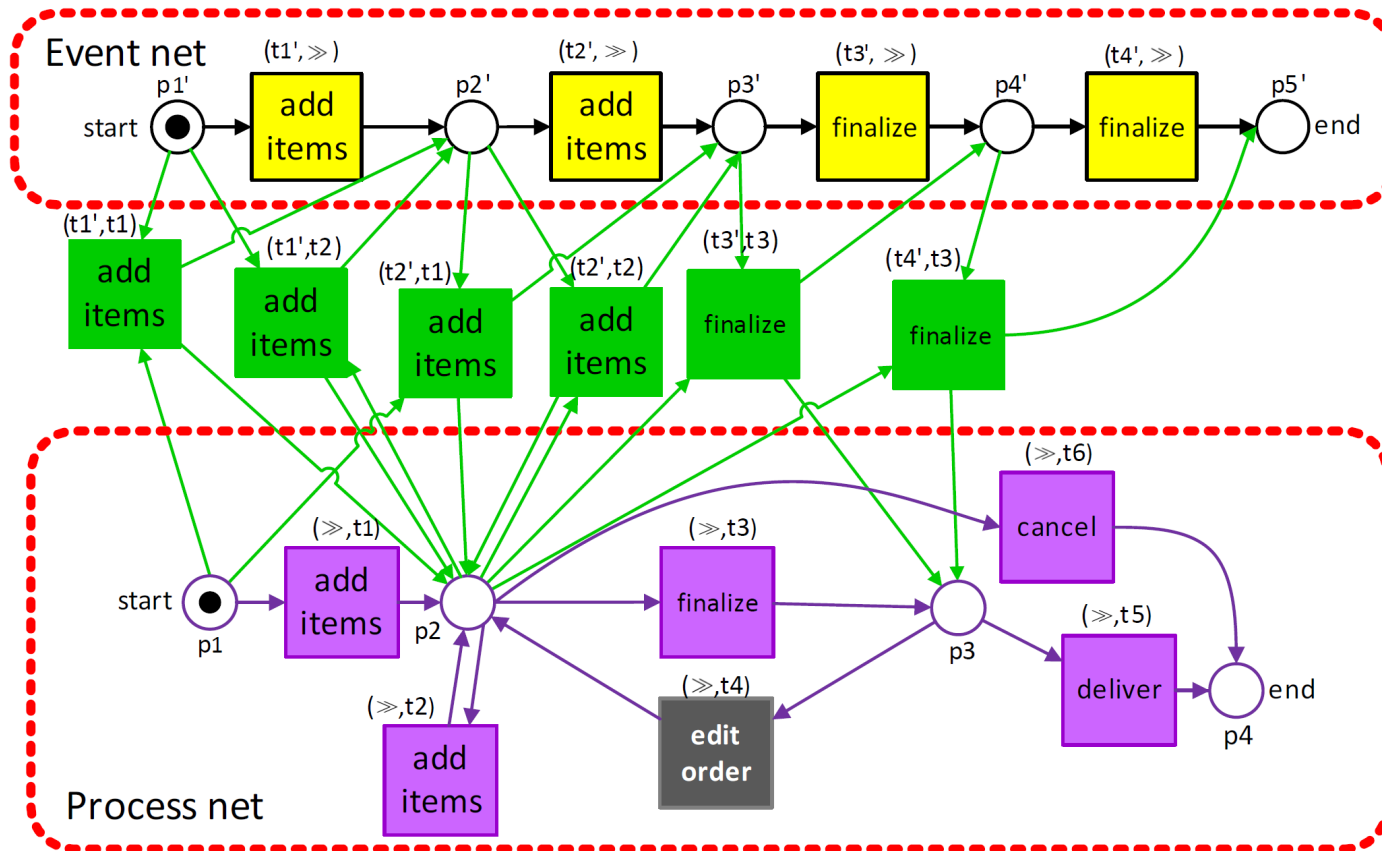
Move on model

Move on model (invisible transitions)

Synchronous move

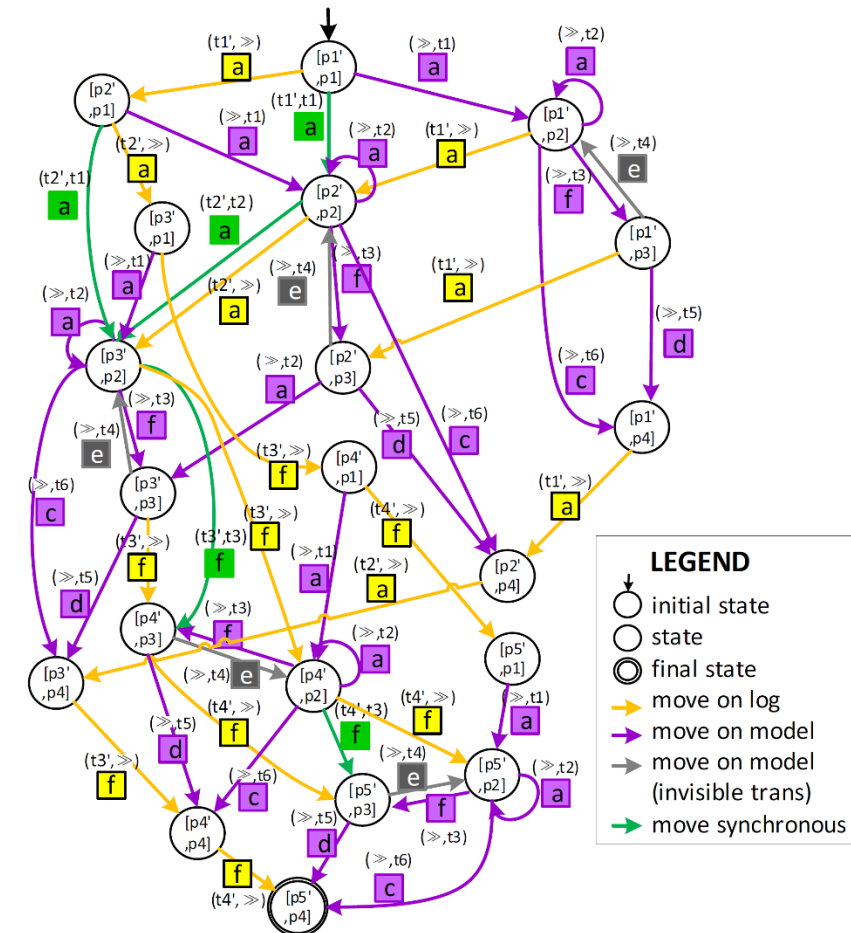
Move on log

# A shortest path problem (in a possibly infinite state space)



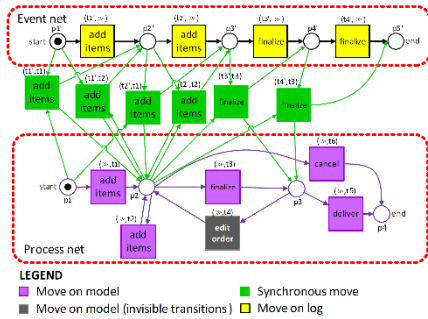
## LEGEND

- Move on model Synchronous move
- Move on model (invisible transitions) Move on log



add items	add items	finalize	$\gg$	finalize	$\gg$
add items	add items	finalize	edit order	finalize	deliver
$t_1$	$t_2$	$t_3$	$t_4$	$t_3$	$t_5$

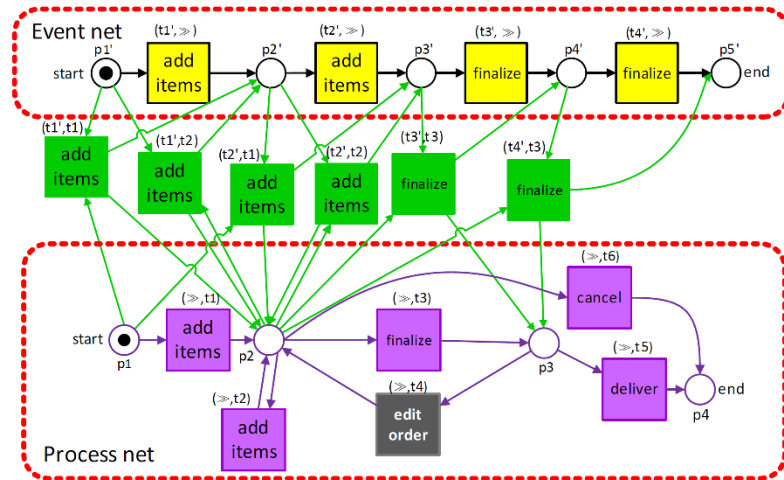
# Marking equation of the product net



$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} t1' \\ t2' \\ t3' \\ t4' \\ t1't1 \\ t1't2 \\ t2't1 \\ t2't2 \\ t3't3 \\ t4't3 \\ t1 \\ t2 \\ t3 \\ t4 \\ t5 \\ t6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

Any path in the product net from start to end is a solution of this system of inequalities

# Efficiently computing optimal alignments



## LEGEND

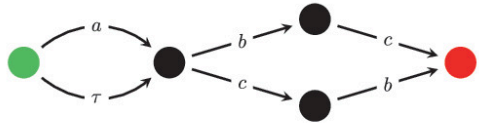
- Move on model
- Move on model (invisible transitions)
- Synchronous move
- Move on log

- The marking equation helps to limit the search space (cut off paths not leading to the end).
- Provides very effective lower bounds to reach the end (exploited by e.g. A\*)

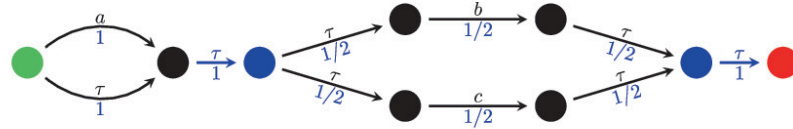
$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & -1 & 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} t1' \\ t2' \\ t3' \\ t4' \\ t1't1 \\ t1't2 \\ t2't1 \\ t2't2 \\ t3't3 \\ t4't3 \\ t1 \\ t2 \\ t3 \\ t4 \\ t5 \\ t6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

# Another Optimization-Based Approach

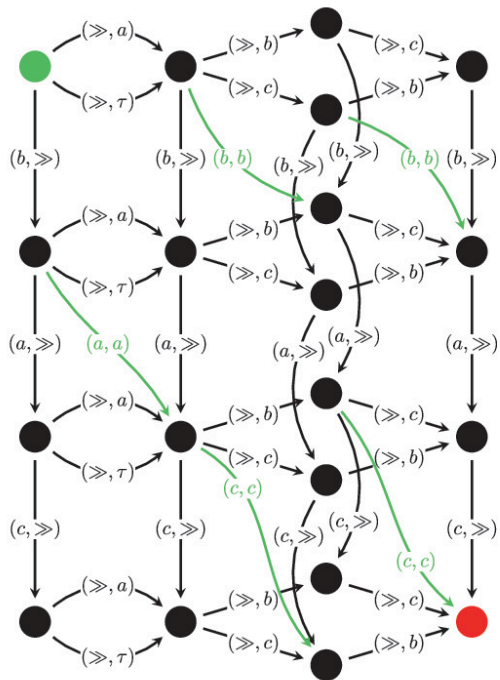
(Based on work with Christopher Schwanen and Wied Pakusa)



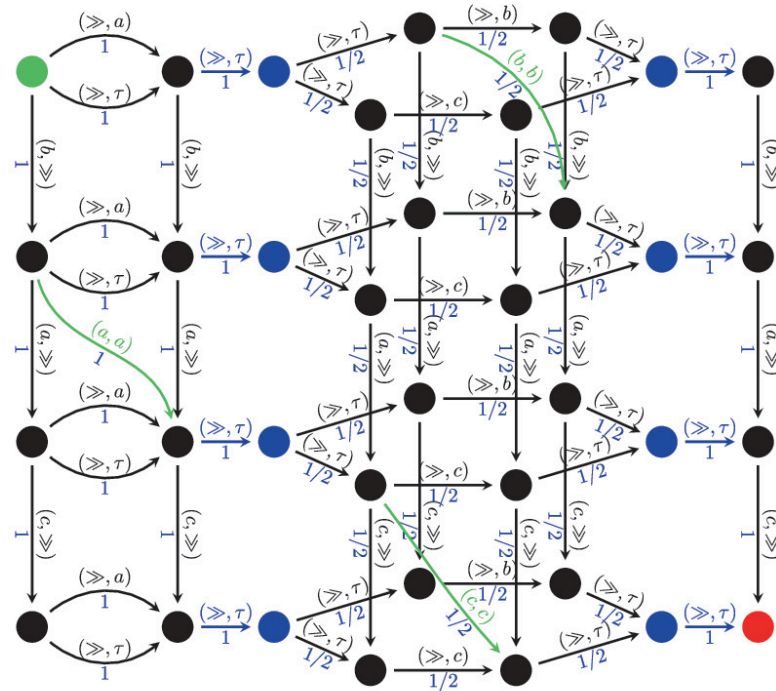
(a) Transition system  $TS(PT)$ .



(b) Process tree network  $\mathcal{N}(PT)$ .



(c) Synchronous product of transition systems  $TS_\sigma$  and  $TS(PT)$ .

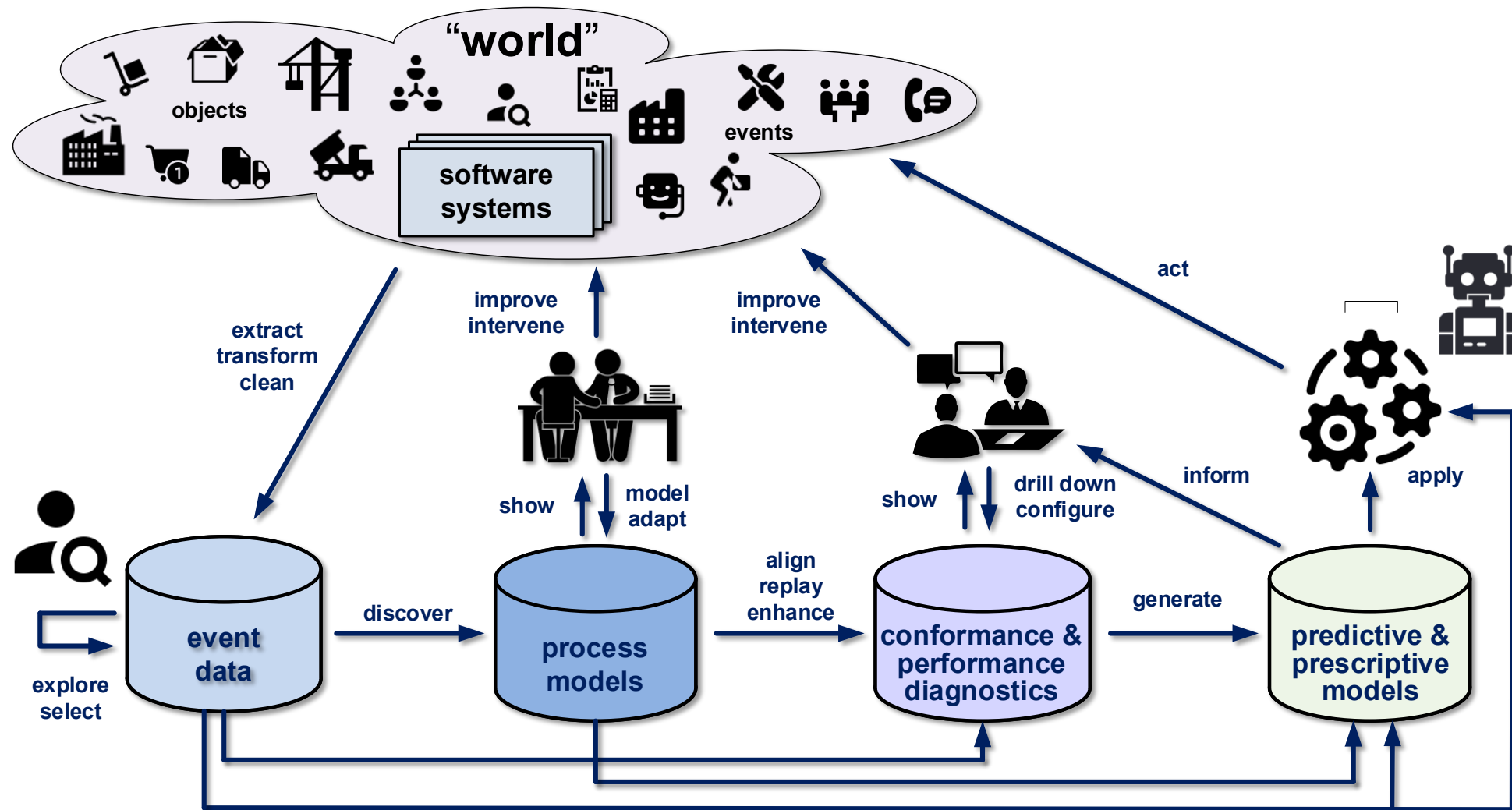


(d) Synchronous network  $\mathcal{N}(\sigma, PT)$ .

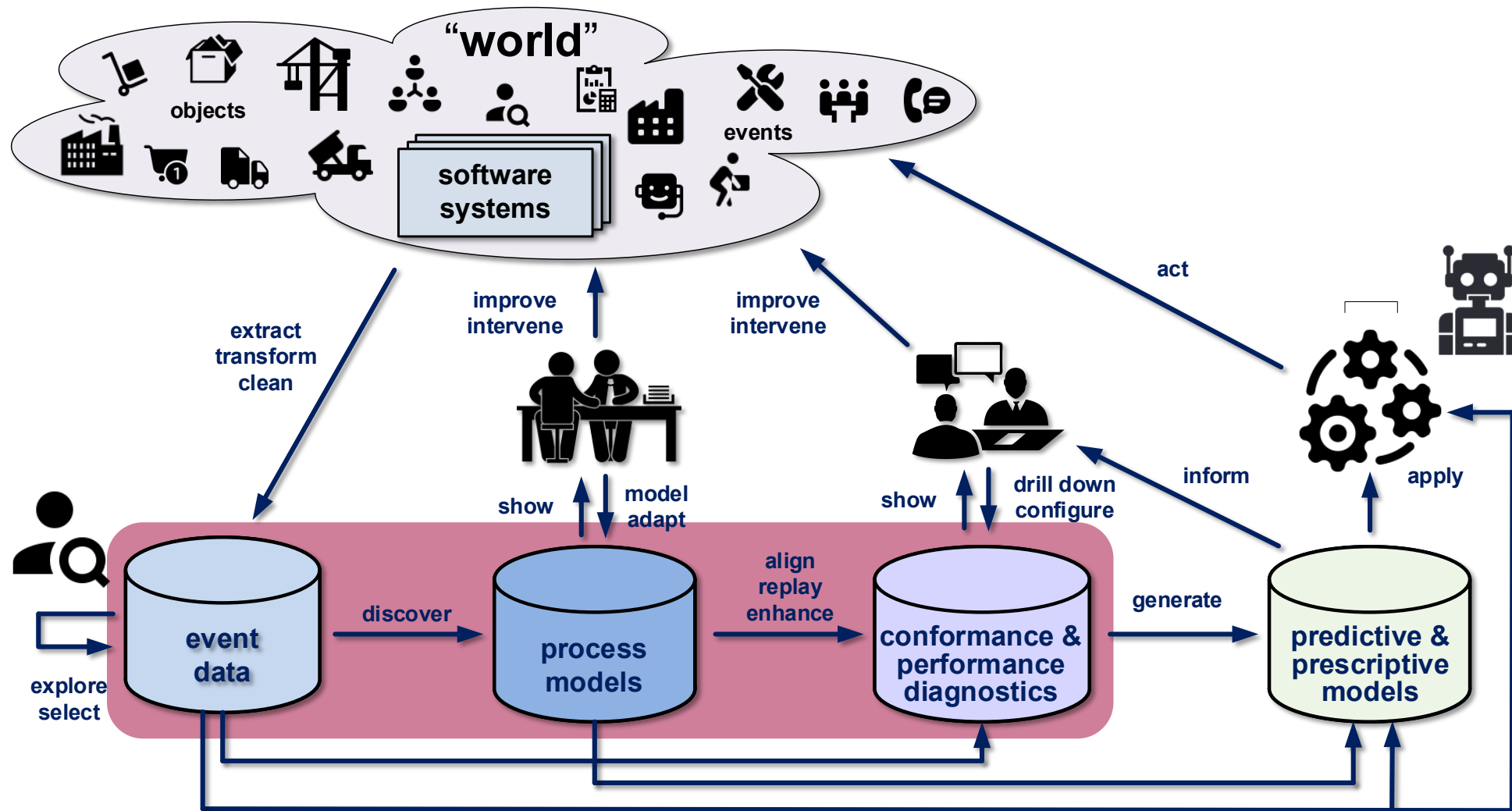
- Alignments on process trees can be expressed as solutions of Mixed Integer Linear Programs (MILP).
- More specifically, we encode the alignment problem as a minimum-cost network flow problem.
- For process trees without parallel executions, our MILP formulation becomes a Linear Program (LP), which can be solved efficiently.
- The approach can be extended to sound free-choice nets.

# How about AI & OR?

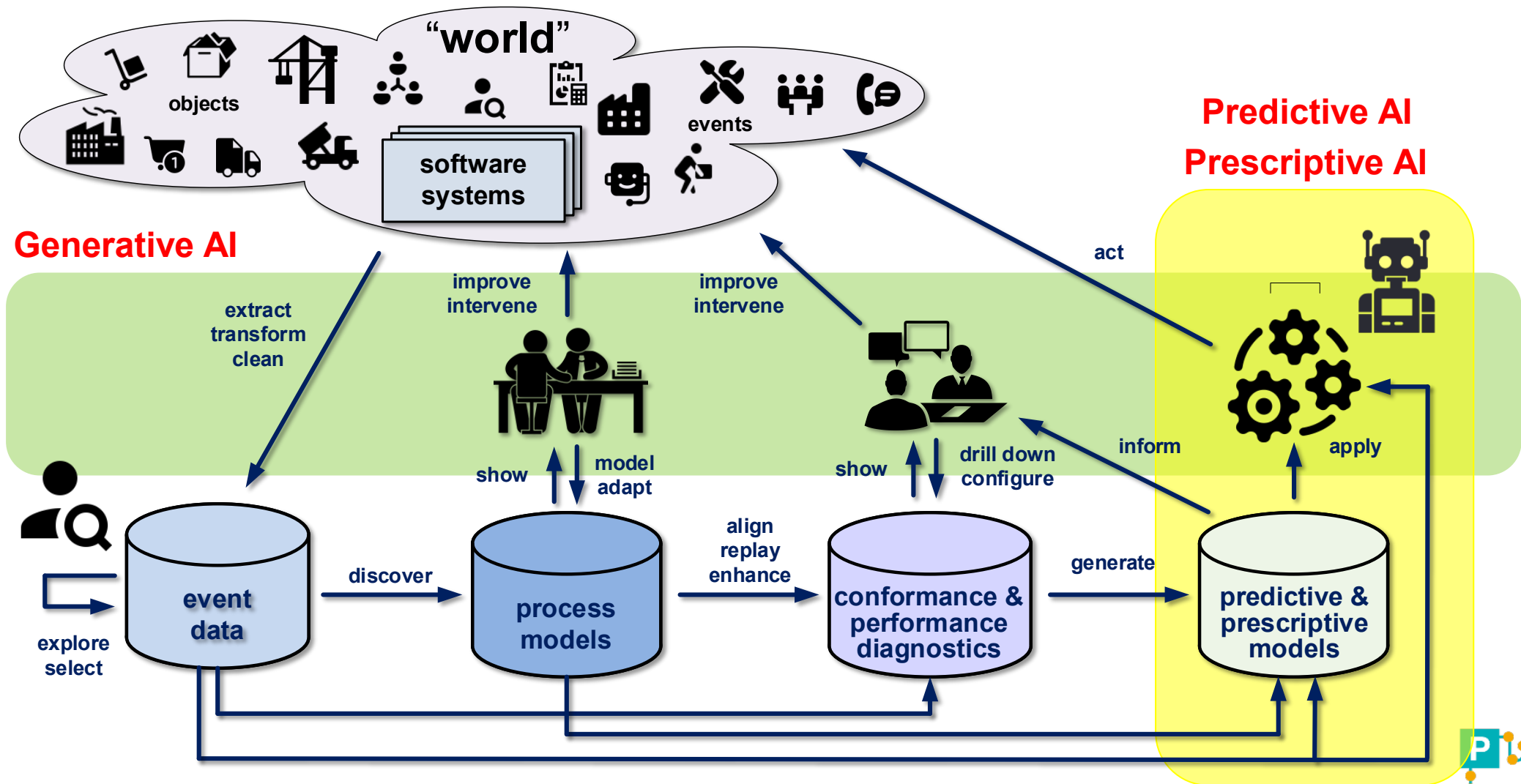
# Refining the Overview of Process Mining



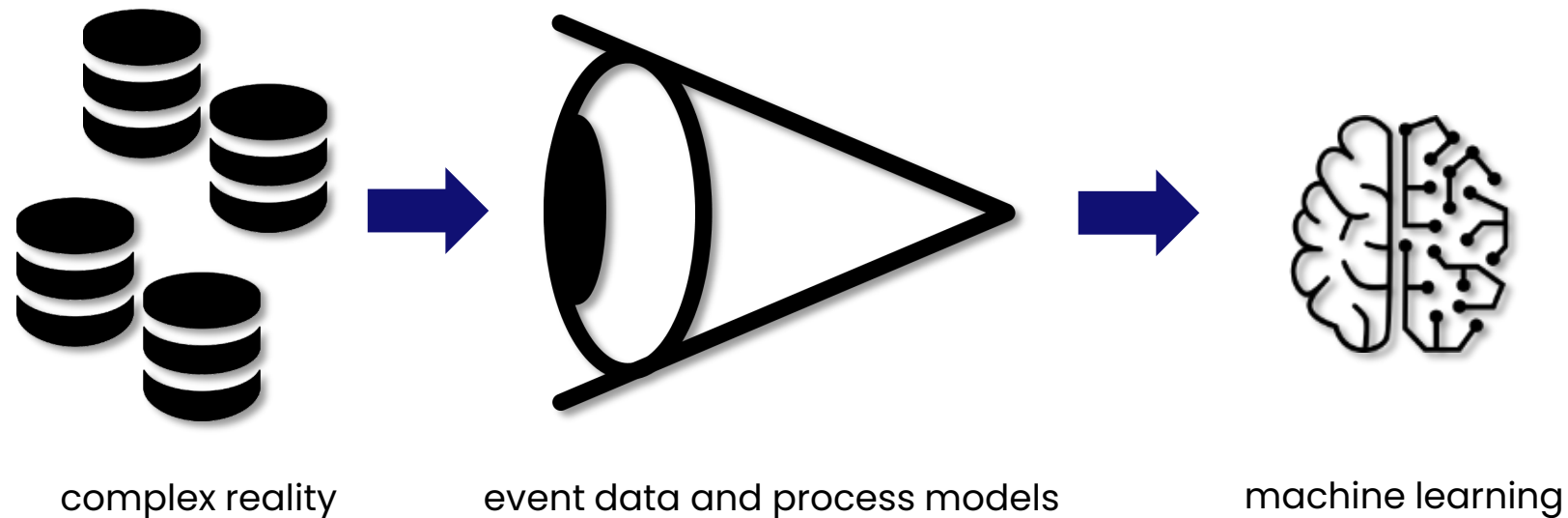
# Focus thus far



# Predictive, Prescriptive, and Generative AI



# Process mining provides the lens to look at enterprise data and processes



# Link to Optimization

# Challenges Optimization

- Involves ad-hoc modeling tailored towards a particular problem.
- No out-of-the-box initial visualization or analysis.
- No standard input representation like in process mining.
- Assumes a “controllable world”, often no feedback loop when the “planned reality” deviates from the “real reality”.



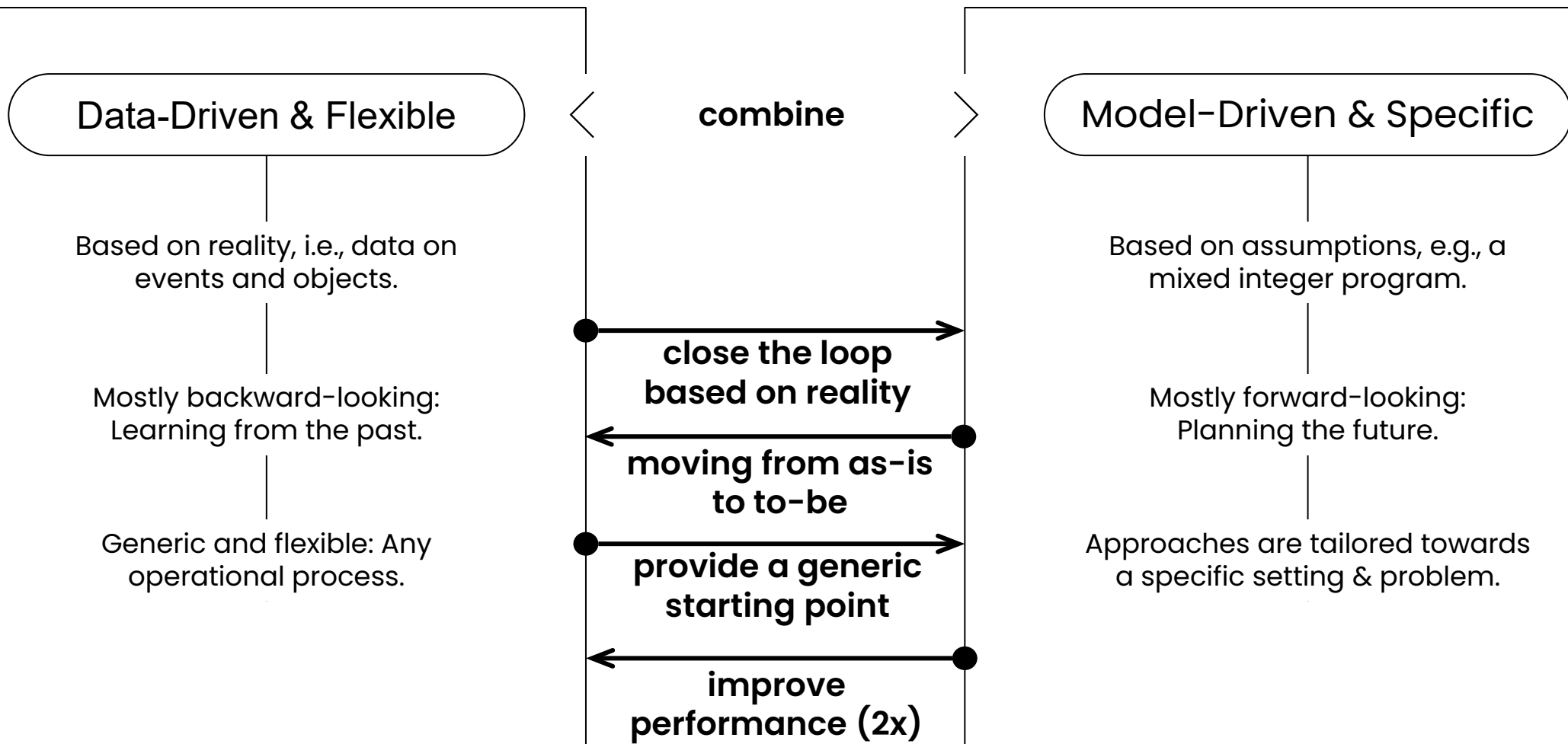
**INFORM**

**celonis**



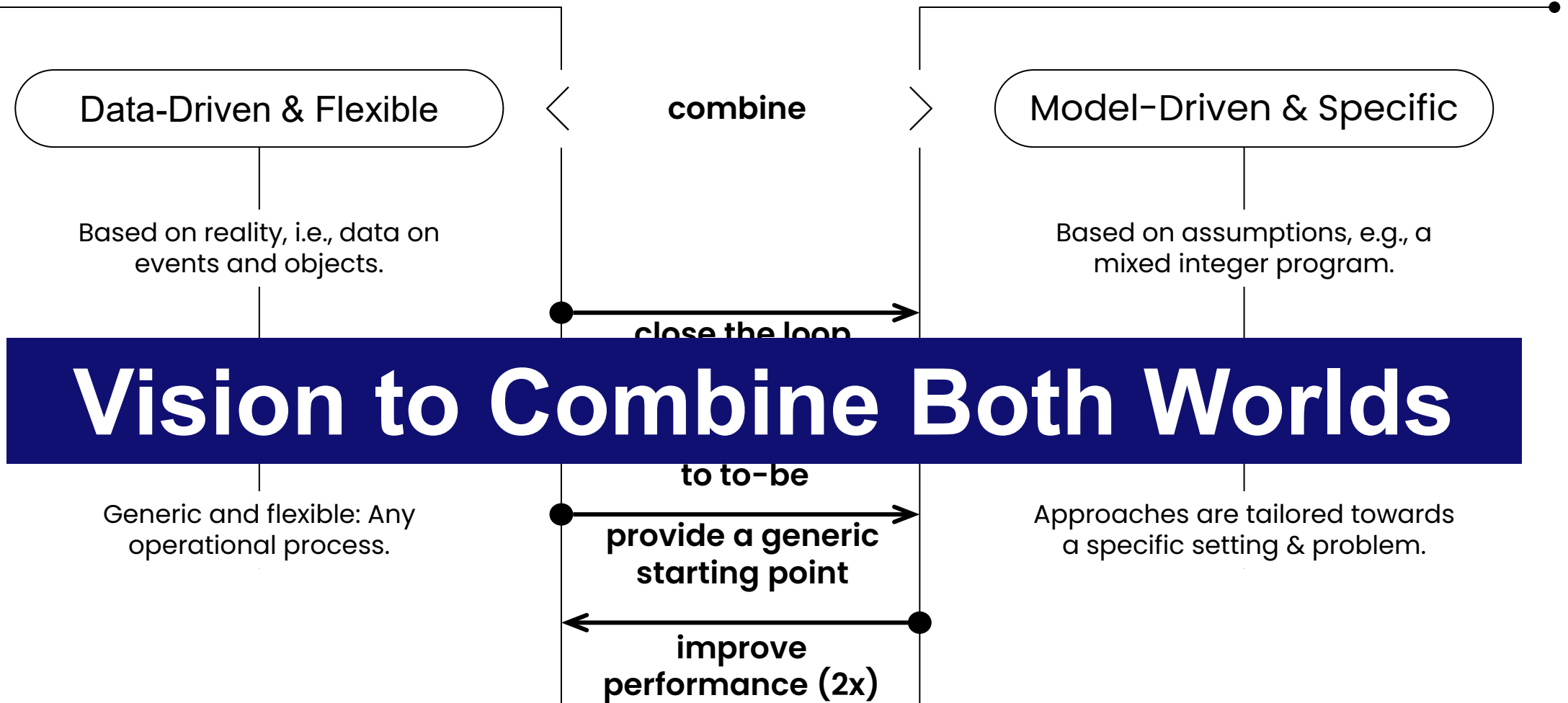
# Process Mining

# Optimization



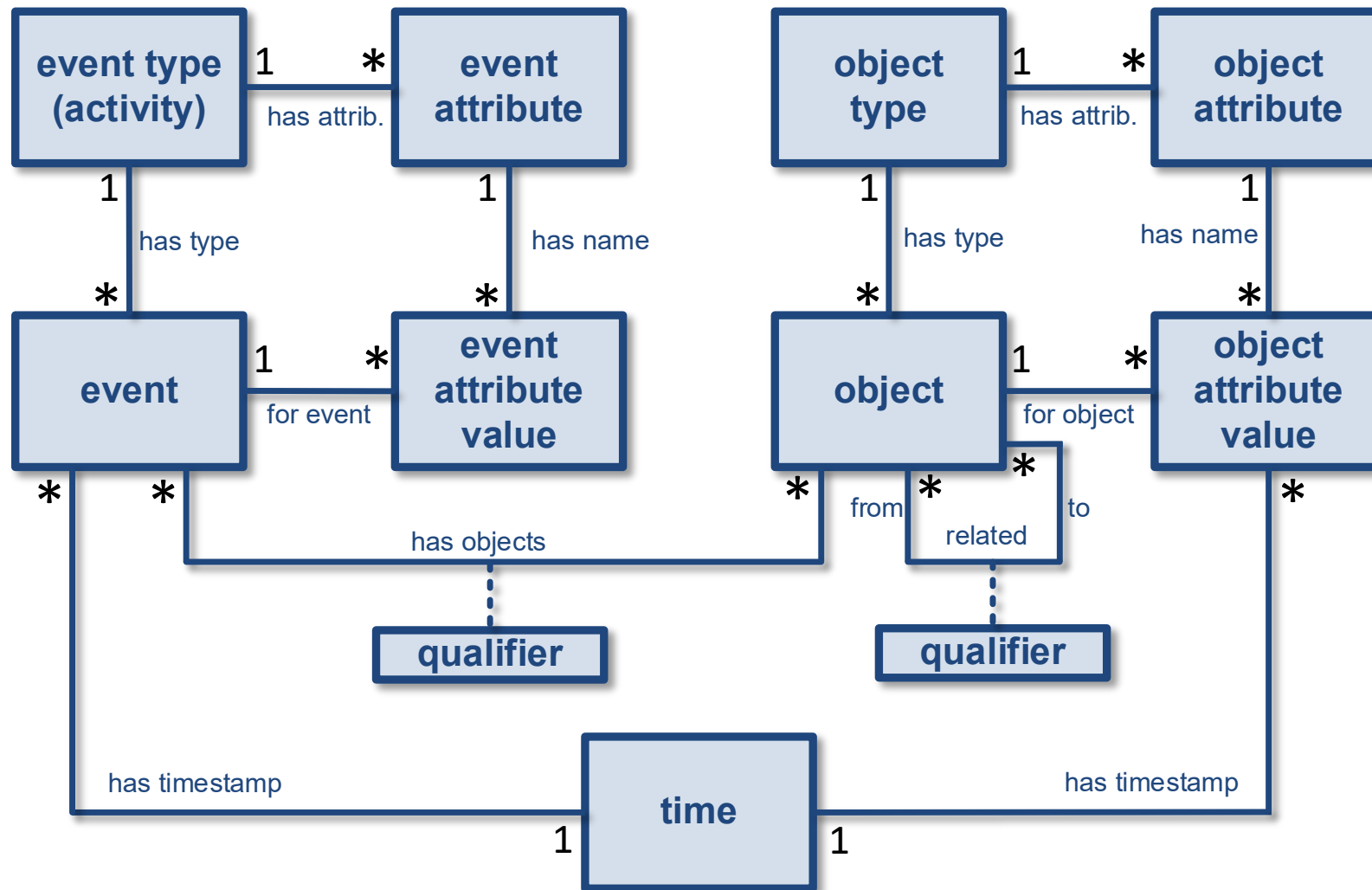
# Process Mining

# Optimization

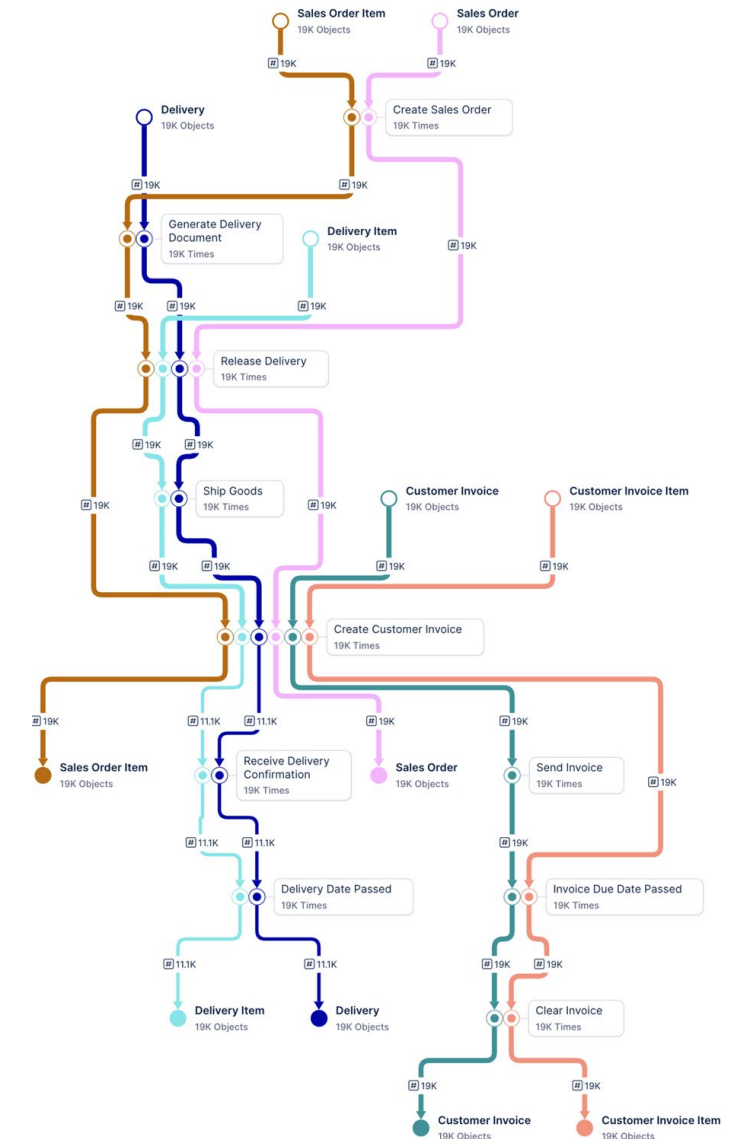


# Object-Centric Event Data (e.g. OCEL 2.0)

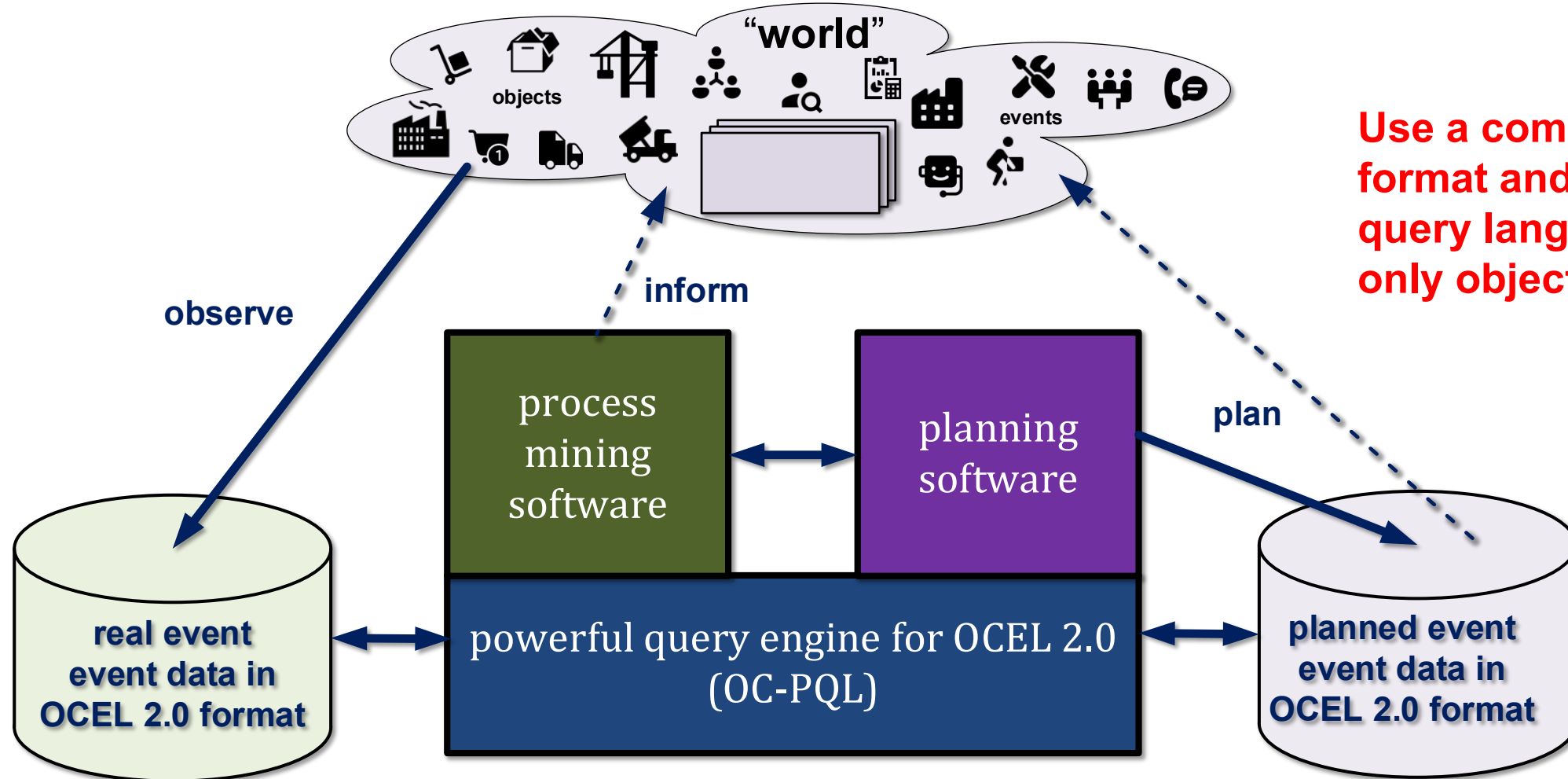
## Have Proven To Be Extremely Powerful



OCEL 2.0 Meta Model



# Use A Common Language For An “Optimized Reality” and the “Real Reality”



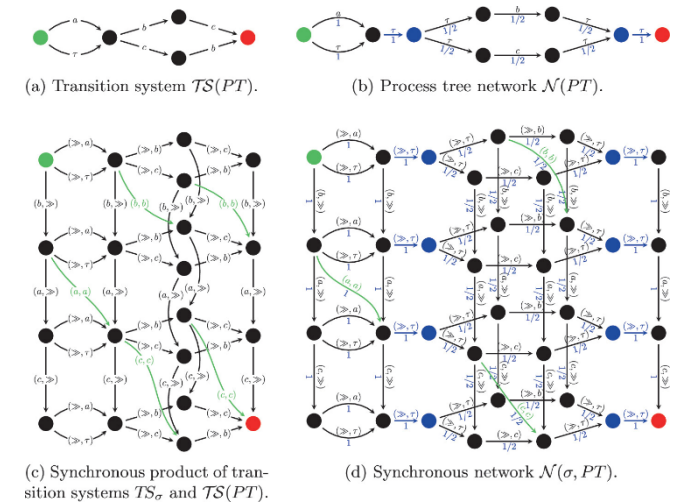
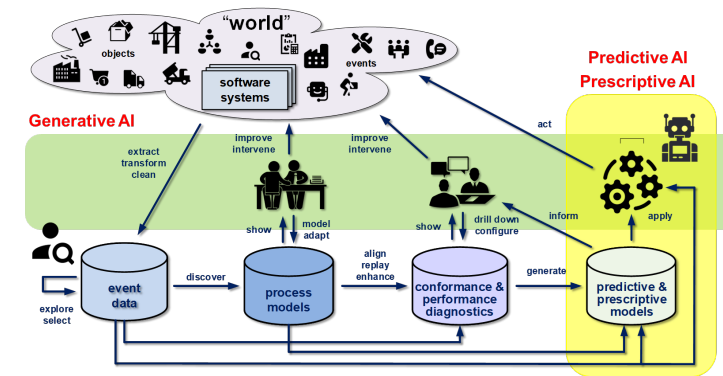
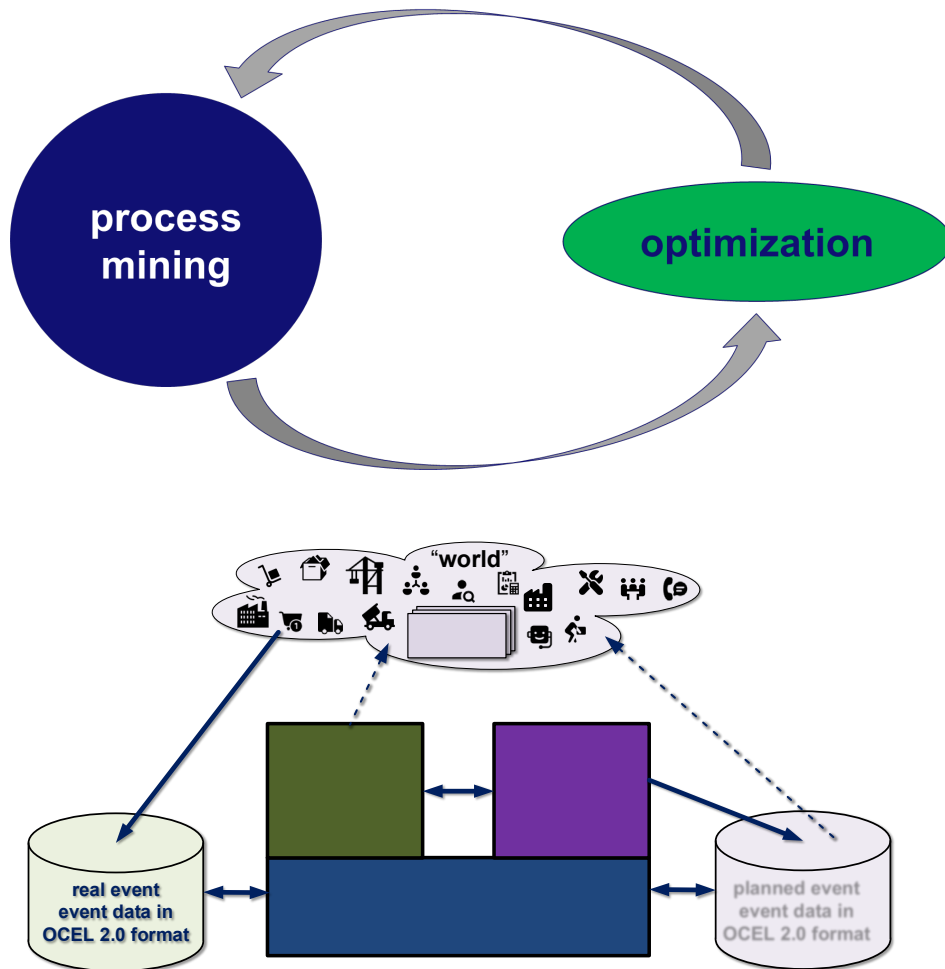
Use a common data format and a common query language using only objects and events

# Topic for collaboration?



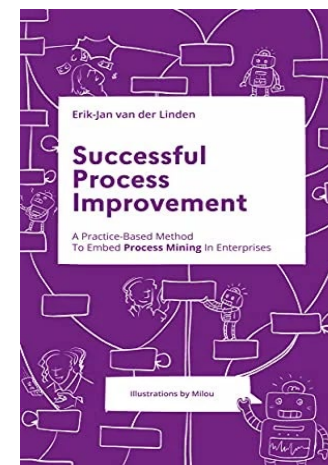
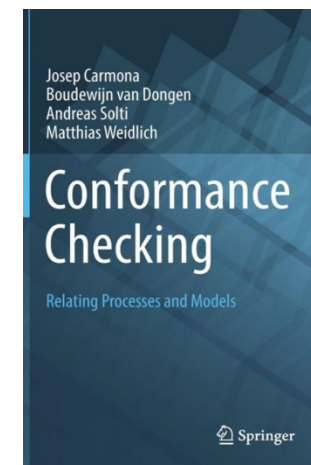
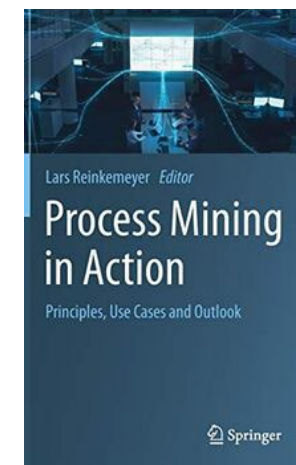
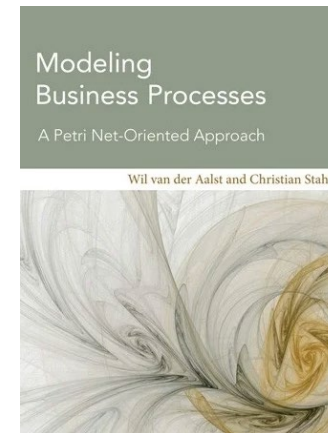
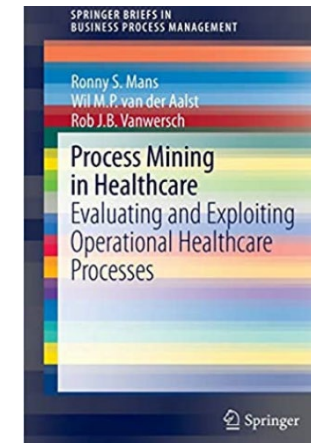
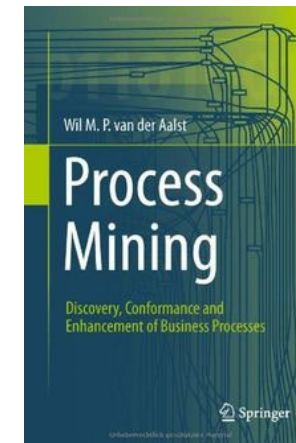
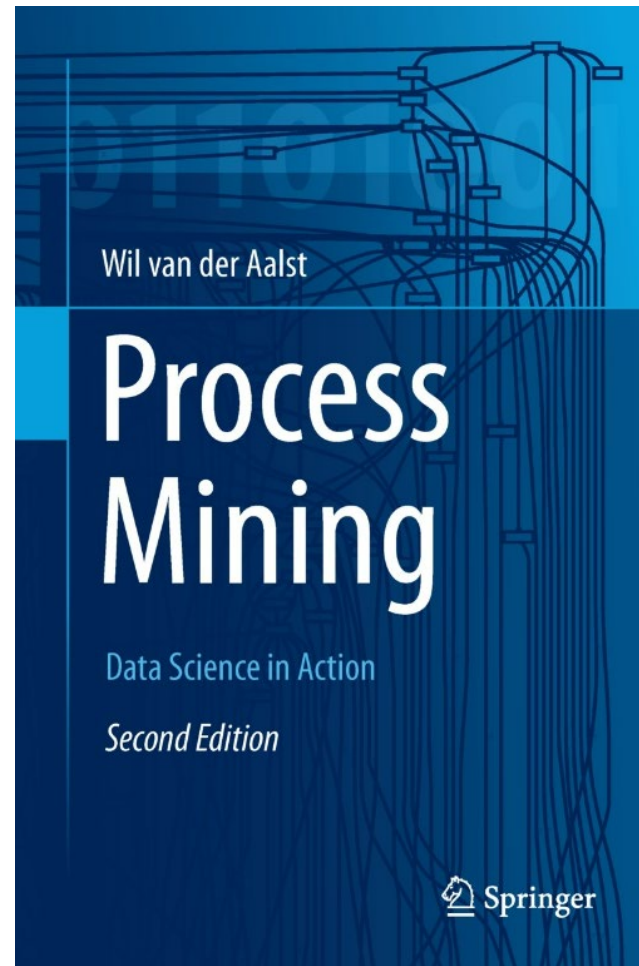
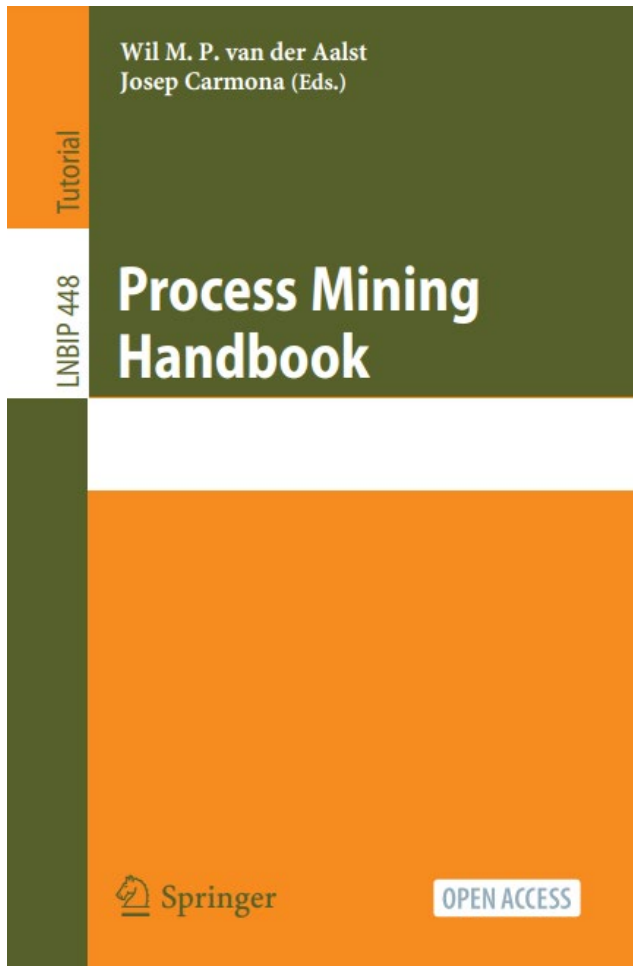
# Conclusion

# Conclusion



# How to learn more?

# Books (not intended to be complete)



# Online courses

- Coursera course  
“**Process Mining: Data science in Action**”
- Three RWTHx edX courses



**coursera**



# Websites

- [www.processmining.org](http://www.processmining.org)
- [www.process-mining-summer-school.org](http://www.process-mining-summer-school.org)
- [www.tf-pm.org](http://www.tf-pm.org)
- [www.promtools.org](http://www.promtools.org)
- [www.celonis.com/academic-signup](http://www.celonis.com/academic-signup)
- [xes-standard.org](http://xes-standard.org)
- [ocel-standard.org](http://ocel-standard.org)
- [www.pads.rwth-aachen.de](http://www.pads.rwth-aachen.de)
- [www.vdaalst.com](http://www.vdaalst.com)

